Net Present Value Analysis for Non-Taxable Entities

Guidelines, Hints and Definitions

(Included in *Take AIM* as the Technical Appendix: Module 5)

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PREFACE

Most of what I think I know about the basics of investment analysis and risk assessment I learned from my friend and mentor, Len Bauer. Len's work and teaching has influenced farm managers and their advisors around the world. I consider it an honour and a privilege to write a short forward for this series of modules on investment analysis.

So, why should managers be interested in this series of investment analysis modules? The answer is because the consequences of poor capital investment decisions can directly determine the financial viability of a business enterprise. One need look no further than the North American hog industry investment boom of the 1980's and 90's and watch the fallout occurring today to see a classic example of capital investment decisions gone wrong. The failure to apply sound management principles invites the market place to solve your management problems for you. The market plays no favorites, treating both big and small businesses the same.

These self-directed learning modules demonstrate the basic tools used in the business world today; they are the language and practice of modern business. My biases on the importance of having a strong understanding of management concepts come from over a decade spent as a researcher and instructor at the University of Alberta blended more recently by several years as manager of a commodity production business.

I have worked with many excellent business managers and if there is a central theme it is this: they distinguish themselves by their knowledge and ability to apply the basic principles of economic decision making and risk management. These modules outline the basic principles and give practical insights, through illustrations and exercises, on how the material can be applied in practical situations.

The following modules lay out the process of analyzing investment decisions. Although the discussion in the modules is restricted to simplified cases, the tools can be applied to any business enterprise. Even if a manager does not use the actual detailed methods in every situation (for example some of the tools contained in the technical appendix) there is power in understanding the proper process for collecting and analyzing the information required for making sound investment decisions. It is impossible to build sound strategies without a solid foundation.

I use these principles in my day to day operations. I strongly encourage managers and those who work with and advise managers in any capacity, to make use of Dr. Len Bauer's work. Today's managers must be able to master these methods and the instructional design provided by Don Bushe makes it easy for busy managers to assimilate the ideas efficiently.

Frank Novak, Managing Director

Alberta Pig Company

Purpose and Objective

The purpose of this document is to provide guidelines, definitions of terms and hints for conducting Net Present Value Analyses. The items are listed as a glossary, in alphabetical order to help locating useful topics. It is hoped that it will provide an easy reference to practitioners interested in conducting economic analyses for contemplated projects.

The author is grateful to Dr. Glen Mumey, Professor Emeritus, University of Alberta and Dr. Larry Bauer, Assistant Professor, Memorial University of Newfoundland for their helpful insights on this important topic, both written and verbal. Any errors that might remain in the document are the responsibility of the author.

Leonard Bauer
October 8, 2003

A Note to accompany the *Take AIM* version

While not really an instructional module, this set of guidelines, definitions, hints and formulae was prepared by Len Bauer in 2003 primarily for the members of the Board of Directors of the Good Samaritan Society. He felt strongly that those making major capital decisions should be aware of The Present Value of a Future Worth – in other words, Net Present Value. That is why the title of the document refers to “Non-Taxable Entities”. We felt that the information, formulae, definitions and hints contained more than make up for the ‘non-taxable’ parts of the title which is why it is included here without major change.

Don Bushe, 2005

List of Topics

Guidelines, Hints and Definitions

Amortization – definition:
The term amortization is used to describe two activities, a) discharging a debt and b) depreciating an asset.

a) The discharging of a debt (e.g. a mortgage) through (usually equal) periodic payments of principal and interest.

b) Also the depreciation of an asset through wear or obsolescence. The value of a fixed asset purchased by a company is not charged in full to its profit and loss account in the year in which it is purchased. Instead it is amortized over its useful life in the accounts, i.e. only a certain portion of its cost is charged to the profit and loss account each year. *Source: The Macmillan Encyclopedia 2001, © Market House Books Ltd 2000.*

Amortization – formulae:
There are a number of formulae useful in applying the amortization principles. These include a) the periodic payment formula, b) the remaining principal formula, c) the amount of principal payable in a specific period formula, and d) the amount of interest payable in a specific period formula. In all cases the period may be but is not necessarily annual. In all cases it is important that the period conforms to the interest rate and that the effective rate is used. See also the section on Interest Rates – compounding periods.

a) Periodic Payment Formula:
The periodic payment of equal amounts to discharge a debt is calculated as follows.

$$A = P\left[\frac{i(i+1)^n}{(1+i)^n - 1}\right]$$

Here (A) is the periodic payment needed to retire a loan of (P) dollars over (n) periods at (i) per cent per period. *See also the present value of an annuity formula .Also see Table A.6 in the appendix for amortization factors.*

b) Principal Remaining at End of a Specific Period:
Sometimes it is important to know the amount of outstanding principal at the end of a particular period. This is readily calculated as follows.

$$L_k = P\left[\frac{(1+i)^n - (1+i)^k}{(1+i)^n - 1}\right] = A\left[\frac{(1+i)^{n-k} - 1}{i(1+i)^{n-k}}\right]$$

Here (L) refers to the amount of principal remaining after (k) of (n) payments made on an original principal of (P) dollars requiring (A) dollars per period at a rate of (i) per cent per period.

c) Principal Payable in a Specific Period:
If one wishes to know the amount of principal payable in a specific period one can use the following formula.

$$R_k = Pi\left[\frac{(1+i)^{k-1}}{(1+i)^n - 1}\right]$$

Here (R) refers to the amount of principal payable in period (k) of (n) periods on an original principal of (P) dollars at a rate of (i) per cent per period.

d) Interest Payable in a Specific Period:
Should one wish to know the amount of interest owing in a specific period one can use the following method.

$$I_k = Pi\left[\frac{(1+i)^n - (1+i)^{k-1}}{(1+i)^n - 1}\right]$$

Here (I) refers to the amount of interest owing in period (k) of (n) periods on an original principal of (P) dollars at a rate of (i) per cent per period.

Annuity – definition:
A stream of equal payments to an individual, such as to a retiree, that occur at predetermined intervals (that is, monthly or annually). The payments may continue for a fixed period or for a contingent period, such as for the recipient's lifetime. Although annuities are most often associated with insurance companies and retirement programs, the payment of interest to a bondholder is also an example of an annuity. *Source of definition: Wall Street Words: An Essential A to Z Guide for Today's Investor by David L. Scott © 1977, 1998 by Houghton Mifflin Company.*

Annuity – formulae:
Two formulae are needed for calculating the quantities associated with annuities, a) the future value of an annuity, and b) the present value of an annuity.

a) The Future Value of an Annuity:
The future value of an annuity is the amount to which a stream of equal periodic payments will accumulate if made at the end of the period. This amount is calculated as follows. *See also Table A.3 Amount of Annuity in the Appendix.*

$$FV = A\left[\frac{(1+i)^n - 1}{i}\right]$$

Here (FV) is the future value, (A) is the annual payment made at the end of each year over (n) years at (i) per cent per annum.

b) The Present Value of an Annuity:
The present value of an annuity is the value at present of a stream of equal periodic payments received at the end of each period. This amount is calculated as follows.

$$PV = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

Here (PV) is the present value, (A) is the annual payment made at the end of each year over (n) years at (i) per cent per annum. *See also Table A.5 Present Value of an Annuity in the Appendix.*

The present value of an annuity formula is widely used by appraisers following the income approach to establishing the value of assets having a finite life. If the asset being valued has a perpetual life, as in the case of land, the present value of a perpetual annuity is used. The formula for the perpetual case is as follows.

$$PV = A\left[\frac{1}{i}\right]$$

Cash Flows – debt repayment:
Normally debt financing costs and payments are irrelevant in determining the economic worth of a project. Remember the purpose of Net Present Value Analysis is to compare the economic value of the project to its cost (i.e. to the capital outlay). Generally speaking the project's value is unaffected by whether it is undertaken with borrowed funds, (e.g. mortgages), or from company surpluses. Take an example from personal finance where you might consider the purchase of a car. The value of the car usually remains unaffected by whether you decide to borrow from the bank to finance the deal or whether you cash in another investment for this purpose. So it is with assessing the economic worth of an investment. Including mortgage payments as part of the cash flows causes tremendous complication and puts one in danger of double counting.

This is not to say that financing is unimportant. It is important, but it's a problem separate from the question answered by Net Present Value Analysis. The logical progression in deciding capital investment issues is to first determine the economic value of the project and then, if the value is sufficient to justify going ahead, examine financing alternatives.

There is an exception where financing may become important during the Net Present Value analysis. Suppose you are able to get subsidized credit for a specific project but this concession is not available for another project. In this case you may enjoy a special benefit directly attached to the project. In such a case the benefit should be included in the analysis. An example illustrates this point.

Suppose you are eligible for a $100,000 mortgage loan at 5.0% per annum over 10 years but only if you undertake a specific project. A commercial loan for the same face amount over the same time frame would cost 7.5% per annum. The Present Value of this benefit i.e. of the reduced rate is then directly attributable to the project and should be included, especially if we are comparing two mutually exclusive investment opportunities.

The annual benefit would be the difference in yearly mortgage payments. The annual payment to retire the mortgage debt is readily calculated with the amortization formula.

A loan at 7.5% requires an annual payment of $14,568.59 while at the 5% rate this is $12,950.46. The annual saving of $1,618.14 would be considered a direct benefit to the project and should be added in as a cash flow element.

Cash Flows – expected values:
When preparing cash flow estimates for Net Present Value Analyses we may be tempted to use a pessimistic forecast as a method for recognizing risk. The conventional approach is to add a risk premium to the discount rate as compensation for risk. If we choose a pessimistic estimate for the cash flow and add a risk premium to the discount rate we will have double counted and the analysis will be biased. Consequently we may reject otherwise attractive projects as a result of the biased analysis. Hence the rule is to adjust the discount rate for risk and use "expected cash flows" for the analysis. (See also the section on expected value).

Cash Flows – nominal:
Nominal cash flows are stated in the dollars of the time period when they are expected to occur. In other words, inflation is embedded in these values. Cash flows will most likely be expressed in real terms as a result of usual estimating procedures. Not all components

of the cash flow components necessarily inflate at the same rate. Thus, if rates of inflation vary substantially from one component of the cash flow stream to another (e.g. wage rates might inflate at a different rate from that applicable to supplies or utilities) it becomes necessary to express cash flows in nominal terms.

If the cash flows are stated in nominal terms it is crucial that you use the nominal discount rate when completing the Net Present Value Analysis.

Cash Flows – real:
Real cash flows are valued at today's prices. In other words expectations of inflation are not embedded in the estimates. Usually, when cash flows are estimated, they are estimated in real terms. In all likelihood you would base the estimates on current year prices and then use this expectation to extrapolate ahead into the future. As a result, your projections would be in current dollars, (i.e. expressed in the dollars of this year).

If the cash flows are stated in real terms it is crucial that you use the real discount rate when completing the Net Present Value Analysis.

Cash Flows – tests for relevance:
Relevant cash flows are those that will pertain should the project be implemented. Only those costs that are incremental to the project are considered.

For example, suppose your organization is planning to offer a new service. As a result additional people must be hired and trained. As well it will be necessary to purchase certain supplies. Because the new service is under patent, permission must be acquired at a cost. This cost must be paid up front but will be amortized over the next ten years. There is sufficient space in your existing facilities however some renovations must be undertaken. As a result financing, secured by a mortgage, will be necessary. By the way, this area of the building is already being serviced by heat and power so there will be no additional utility costs. Also there is sufficient capacity in the administrative unit, [i.e. accounting, invoice preparation, payroll, etc.] to accommodate the new service. Which of these elements are relevant in preparing the cash flow forecast for the new project? See the table for summary.

Assessing the Relevance of Cash Flow Elements

Element	Relevant	Notes
Salaries	Yes	Direct result of project
Staff Training	Yes	Direct result of project
Supplies	Yes	Direct result of project
Patent Acquisition	Yes	Direct result of project
Patent Amortization	No	Paid up front – no annual cash amount
Renovations	Yes	Direct result of project
Mortgage Payments	No	Financing irrelevant to decision – see financing section
Utilities	No	No extra cost due to project
Administration	No	No extra cost due to project

A special point about fixed costs should be noted here. A single expansion project may be of relatively small proportion to the overall asset base of the organization so that there is sufficient capacity in the administrative unit. However, as more and more projects are added, this capacity may become strained with the consequence of increased administrative costs. For example more staff may be needed and existing employees may expect higher salaries commensurate with increased administrative responsibilities.

Discount Rate – conceptual framework:

The discount rate (r_D) is that rate which will make an informed investor indifferent between an amount of money today, i.e. a present value (*PV*) and an amount (*n*) periods in the future i.e. a future value (*FV*).

$$PV = \frac{FV}{(1+r_D)^n}$$

Conceptually the discount rate is composed of three factors: a) the pure time value of money; b) compensation for risk in the investment; and c) compensation for potential loss in purchasing power of the dollar, i.e. inflation.

a) The pure time value of money:

The pure time value [i.e. the cost of postponing present consumption to the future] may be represented by the risk-free rate embodied in the rate paid on Government Bonds, e.g. Government of Canada Treasury Bills. Here we express the nominal risk-free rate as ($\tilde{r}_F$).

b) Compensation for risk in the investment:

Generally speaking a premium is added to the risk free rate to account for the risk inherent in the project where the risk free rate is represented by Government Bonds. A good starting place for thinking about risk premia is the common stock market, for example the S&P/TSX Composite or the Dow Jones Industrials. The underlying notion is that investors need a premium to entice them into the purchase of Common Stocks instead of Government Bonds. Accordingly the relationship can be expressed as

$$\tilde{r}_M = \tilde{r}_F + (\tilde{r}_M - \tilde{r}_F)\ .$$

Here ($\tilde{r}_M$) is the nominal return on a well diversified market portfolio, for example the S&P/TSX Composite, and ($\tilde{r}_F$) is the nominal risk free rate. The quantity $(\tilde{r}_M - \tilde{r}_F)$ is the nominal risk premium. The relationship of a particular project is expressed as

$$\tilde{r}_P = \tilde{r}_F + \tilde{\beta}_P(\tilde{r}_M - \tilde{r}_F)\ .$$

Here ($\tilde{r}_P$) is the nominal rate of return required on the project, (i.e. the discount rate required for the project) and ($\tilde{\beta}_P$), called the Beta, is an index expressing the relationship of the project's risk to the market generally. If the index is greater than one (i.e. $\tilde{\beta}_P$ >1) the project demands a greater premium than what is required to interest an investor in the Stock Market. Should the index be less than one (i.e. $\tilde{\beta}_P$ <1) a smaller premium will suffice.

c) Compensation for inflation:

In the above formulation all rates have been expressed in nominal terms, i.e. as they are found in the market place of the day. Frequently it becomes necessary to express the discount rate in real terms. Nominal rates are readily converted to real rates by deflating them. This compensation for potential loss in the purchasing power of the dollar is Compensation for Inflation. The process of converting nominal rates to real rates is expressed as

$$r_P = \frac{(1+\tilde{r}_P)}{(1+r_I)} - 1 .$$

Here (r_P) is the real discount rate for the project, ($\tilde{r}_P$) the nominal discount rate for the project and (r_I) the expected rate of inflation.

Canadian Average Annual Rates of Return (1926 – 1981)

Portfolio	Nominal Return	Real Return	Risk Premia
Common Stocks	11.7 %	8.3 %	8.1 %
Corporate Bonds	4.1 %	-0.3 %	0.5 %
Government Bonds	3.7 %	0.8 %	0.1 %
Treasury Bills	3.6 %	0.5 %	0.0 %

Source: adapted from various sources quoted by Richard Brealy, Stewart Myers, Gordon Sick and Robert Whaley, Principles of Corporate Finance, First Canadian Edition. McGraw Hill Ryerson, 1986, pp 127, 132. Data cover 1926 – 1981 except for Corporate Bonds 1949 - 1981

Discount Rate – nominal:

The nominal discount rate is a rate determined from its various nominal or market components [i.e. prime lending rates, risk-free rates, risk premia].

Discount Rate – not-for-profit situations:

What discount rate should be used for a not-for-profit situation; should it be lower than the rate a profit oriented situation would require? To answer that question one needs to look at the reason for NPV analysis in the first place. NPV analysis is done to determine the economic value of a project. Why would the economic value of the project change just because it was being contemplated by a not-for-profit organization? There doesn't seem to be a good answer as to why it would. It may be that a not-for-profit organization might undertake a project with a low economic value that would be rejected by a commercial concern because of wider community objectives, but even so, knowing the economic value seems essential. Consequently, it seems sensible that a not-for-profit organization should derive the discount rate in a manner identical to that used by a commercial concern. A Net Present Value analysis based on a market derived discount rate is what establishes the economic value.

Discount Rate – practical estimation approaches:

Whilst the conceptual basis for establishing the discount rate provides a useful understanding of its components the data required may prove to be elusive, especially in establishing the project Beta (β_P). To assist in the practical task of finding a suitable discount rate we suggest two, albeit subjective approaches. We might use a) subjective Beta values, or b) base it upon the risk premium used by lenders in financing similar projects.

a) Using Subjective Beta Values:

In the absence of hard data one might resort to past experience and assign a value for Beta subjectively. At the time this is written, the risk-free rate (i.e. the yield on one year Treasury Bills) is 2.96%. Over the long term (that is 1926-1981) the risk premium of common stocks over the T-Bill rate has been 8.10%. If we assume, subjectively to be sure but supported by experience, that the project being considered is equal in risk to the stock market we would assign a Beta value of one, ($\beta_P = 1.0$). Consequently this would establish a discount rate of 11.06% (2.96 + 1.0 x 8.10 = 11.06). Were we to assume the project is less risky than the market, say $\beta_P = 0.5$, we would accept a discount rate of 7.01% (2.96 + 0.5 x 8.10 = 7.01). Were we to assume the project to be riskier, say $\beta_P = 1.5$, the rate would be 15.11% (2.96 + 1.5 x 8.10 = 15.11).

b) Basing the Discount Rate on the Lender's Risk Premium:
An acceptable and practical alternative might be to base the rate on the loan rate, i.e. the commercial lending rate applicable for the project. The lender will have incorporated a risk premium into the lending rate however this premium is conceptually different from the risk level assumed by the investor in the project. The lender is mainly concerned with risk of default. The investor is concerned with the potential variance of actual cash flows from those predicted. In most circumstances the investor bears a greater risk than does the lender.

Following this approach the investor might double the premium for risk of default charged by the lender. For example suppose that the lender charges three percentage points above the prime lending rate as the rate at which he is willing to lend to the investor. At the time this is written, the prime lending rate is 4.5% and the rate on five year commercial mortgages is 8.0%, hence a premium of three and a half percentage points. By doubling the premium the investor would require a premium of six points above prime as a fair rate in consideration of the project. In this case the appropriate discount rate would be 11.5%, [4.5 + 2 x 3.5 = 11.5].

The discount rates discussed above for either of the two approaches are, of course, in nominal terms and may need to be deflated into a real rate, depending upon the structure of the analysis problem.

The table summarizes the results of the subjective approaches to deriving suitable discount rates as discussed above.

Market Rates and Discount Rates (as at November 2002)

Current Rate of Inflation	2.30 %	
Risk Premium (1926-1981)	8.10 %	
	Nominal Rates	**Real Rates**
1 year Treasury Bills - Canada	2.96 %	0.65 %
Prime Lending Rate	4.50 %	2.15 %
Residential Mortgage Rate 5 Year Term	7.50 %	5.08 %
Commercial Mortgage Rate 5 Year Term	8.00 %	5.57 %
Discount Rates:		
$\beta_P = 0.5$	7.01 %	4.60 %
$\beta_P = 1.0$	11.06 %	8.56 %
$\beta_P = 1.5$	15.11 %	12.52 %
Double the lenders premium	11.50 %	8.99 %

Source: adapted from Bank of Canada data, personal communication with Alberta Treasury Branches and from other sources.

Discount Rate – real:
The real discount rate is a nominal discount rate with the effects of inflation removed. The process of converting nominal rates to real rates is expressed as

$$r_P = \frac{(1+\tilde{r}_P)}{(1+r_I)} - 1 .$$

Here (r_P) is the real discount rate for the project, ($\tilde{r}_P$) the nominal discount rate for the project and (r_I) the expected rate of inflation.

Expected Values – general:
Expected value is a statistical term referring to the arithmetic average of a sample. Generally speaking a statistically valid sample is taken for which the mean, or arithmetic average is calculated. The underlying premise suggests, that with repeated sampling, one would "expect" to obtain this mean value for the population being sampled. The arithmetic average is computed as

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

In this formulation ($\bar{x}$) is the arithmetic mean (n) is the sample size and (x_i) an observation or element within the sample. The symbol $\sum_{i=1}^{n}$ signifies summation of all elements in the sample from (i=1) through (n).

For example if one observed weights of grade twelve boys in a sample of five boys taken from a class of 25 boys as [45kg, 52kg, 47kg, 49kg, 43kg] the arithmetic mean would be 47.2kg. We might infer from this that average weight of boys in the class is 47.2kg; hence the expected weight is 47.2kg.

Future Value:
The future value is the value of an amount set aside will grow to after compounding at a stipulated rate over a specified number of periods. The relationship between future value and present value is calculated by the following formula.

$$FV = PV(1+i)^n$$

Here (FV) is the future value of (PV), a present value, (n) years away at a rate of (i) per cent. *Please also see the section on Present Value and also Table A.1 "Amount at Compound Interest" in the appendix .*

Handy References:

There are a number of useful links to market information. The Bank of Canada has a "rates and statistics" page http://www.bankofcanada.ca/en/rates.htm. This page provides links to Canadian Interest Rates, United States Interest Rates, Consumer Price Index, An Inflation Calculator, plus numerous others. The Bank also has a glossary of terms at http://www.bankofcanada.ca/en/glossary/glossary.htm which is useful for definitions. More information can be found on the Statistics Canada Web Site at http://www.statcan.ca/start.html. Again follow the links.

In addition to the Web Sites cited there are numerous corporate finance text books covering the topic. Richard Brealy, Stewart Myers, Gordon Sick and Robert Whaley, Principles of Corporate Finance, First Canadian Edition. McGraw Hill Ryerson, 1986 is one such reference.

Inflation – historical rates:

Inflation refers to changes in price levels. These are obtained by tracing the value of goods and services through time and expressed as the value of a basket or bundle of goods and services at selected points in time. In Canada the Consumer Price Index (CPI) is a widely used measure of changes in price levels, namely in the purchasing power of the dollar.

Consumer Price Indices and Rates of Inflation for Selected Years

Year	$\mathbf{CPI}_{(base=1990)}$	$\mathbf{CPI}_{(base=2002)}$	Inflation Rate $r_{I,t}$
1940	10.23	7.99	5.41%
1945	11.63	9.08	1.00%
1950	16.09	12.57	4.14%
1955	18.02	14.07	0.00%
1960	19.72	15.40	0.52%
1965	21.43	16.74	2.57%
1970	25.90	20.23	2.53%
1975	37.53	29.31	10.35%
1980	57.14	44.63	10.74%
1985	80.49	62.86	4.12%
1990	100.00	78.10	4.22%
1995	111.41	87.01	2.24%
1996	113.11	88.34	1.53%
1997	114.93	89.76	1.61%
1998	115.77	90.42	0.74%
1999	118.77	92.76	2.59%
2000	121.96	95.25	2.68%
2001	125.16	97.75	2.62%
2002	128.04	100.00	2.30%

Source: adapted from Bank of Canada data

The table shows Consumer Price Indices (CPI) for selected years. By way of interpretation, a bundle of goods and services available for $100.00 in 1990 would have cost only $57.14 ten years earlier in 1980. A decade later, in 2000, this bundle would have cost $121.96.

The rate of inflation ($r_{I,t}$), which measures the annual change in the CPI for the year (t), is computed as

$$r_{I,t} = \frac{(CPI_t - CPI_{t-1})}{CPI_{t-1}}$$

The subscript (t) refers to the current time period (i.e. the current year) and ($t-1$) to the previous period. For example the rate of inflation for the year 2001 equals

$$r_{I,t=2001} = \frac{(CPI_{2001} - CPI_{2000})}{CPI_{2000}} = \frac{(125.16 - 121.96)}{121.96} = 2.62\% .$$

Inflation – expected rates:
In the table of historical inflation rates we observe the rates over the past eight years, from 1995 to 2002 to have been [2.24%, 1.53%, 1.61%, 0.74%, 2.59%, 2.68%, 2.62%, 2.30%]. The arithmetic average of this series is 2.04%. Under the assumption that the economic performance of the past eight years is a good indicator of the near future, a most naïve assumption to be sure, we conclude the expected rate of inflation to be 2.04%. If other information about economic performance is available we might introduce this into our expectations. *See also the section on Expected Values.*

Interest – definition:

Interest is a charge made by the lender on the borrower as compensation for using borrowed funds. It is based on the amount of outstanding debt. Interest rates are generally expressed as a per cent per period, normally per annum.

Interest Rates – annual contract rate:
The contract rate of interest is that stated in the loan agreement. For example the phrase "ten per cent per annum compounded semi-annually not in advance" appearing on a mortgage agreement declares the contract rate to be ten per cent. The contract rate may differ from the effective rate depending upon the frequency of compounding. *See also the section on Interest Rates – compounding periods.*

Interest Rates – annual effective rate:
The effective rate of interest is the rate actually payable by the borrower on an annual basis. It may differ from the contract rate depending upon the frequency of compounding. For example, suppose the contract rate is ten per cent and the compounding period is six months [i.e. semi-annually]. What then is the effective rate?

The process is to calculate half the interest every six months. In other words, the half yearly or semi-annual contract rate is five per cent [i.e. 10 / 2 = 5]. This means that interest at five per cent is calculated in six months time on the outstanding debt at the start of the year and then again at five per cent in twelve months on the outstanding debt at six months, which now includes the interest calculated at six months. From the lender's perspective one dollar of debt would earn five cents of interest at six months so the borrower would owe $1.05 at that point. In the second six month period [i.e. the second half of the year] the lender would earn another nickel for the original dollar plus a quarter of one cent for interest on the five cents worth of interest earned in the first half. The total interest earned in the year would then be ten and one-quarter cents [i.e. $0.05 + $0.0525 = $0.1025]. This process can be expressed more generally by the following formula.

$$i_e = \left(1 + \frac{i_c}{m}\right)^m - 1$$

Here (i_e) is the effective rate per annum, (i_c) is the contract rate per annum and (m) is number of compounding or conversion periods per annum. *See also the section on Interest Rates – compounding periods.*

Interest Rates – compounding periods:
Although interest rates are normally expressed as a per cent per annum interest may be calculated more frequently than once per year. For example residential mortgage loans in

Canada generally are stated as "a rate per annum compounded semi-annually not in advance." Interest in that case is calculated once every six months.

Interest Rates – monthly effective rate:
The effective monthly rate interest is readily calculated from the annual rate.

$$i_m = (1 + i_e)^{1/12} - 1$$

Here (i_m) is the effective monthly interest rate and (i_e) effective per annum rate. We take the twelfth root of the quantity enclosed in brackets [i.e. for the twelve months in the year] to achieve the desired result. *See also the section on Interest Rates – annual effective rates.*

Investments – their nature:

All investments, whether made by profit oriented entities or by not-for-profit organizations are made today in anticipation of reward in the future; funds are committed to the project upfront with the expectation that these funds will be recovered over time. Because the future can not be foreseen perfectly, there is a risk that what is expected is at odds with what happens. Errors will be made estimating annual cash flows and, to a lesser extent, in estimating the initial investment. There is the risk that actual cash flows in the future will be inadequate to justify the initial capital outlay. On the other hand there is the risk that attractive investments will be passed by because net operating cash flows are underestimated. Net Present Value (NPV) analysis seeks to minimize these risks.

Net Present Value Analysis – components:

The Net Present Value process requires that attention be paid to each of the following components. The components below are ones that would be found in projects involving care for the frail and elderly. While they refer to facilities, projects, and services that would be provided by a Society or Group structured within the community to deliver these needs, they provide an excellent illustration of the NPV process.

c) **The Time Horizon:** The analysis must consider the life span of the project. The physical attributes of the building, the demographics of the region are but two factors affecting the project's time horizon. In all likelihood the project's life will (should) exceed the length of financing e.g. of mortgages and loan agreements.

d) **Initial Outlay of Funds:** The initial outlay of funds is the amount required to fund the expansion. This is likely the least risky of the cash flow estimates because it is rather immediate and normally fixed by contract with the construction firm.

e) **Operating Receipts:** The receipts for projects come from two main sources, from local health regions as a consequence of providing insured services and from rents paid by the residents themselves. The receipts derived from health authorities are subject to government policy which can change from time to time. Rents paid by residents are subject to economic conditions and to social policy. In short, the estimate of receipts is subject to error.

f) **Operating Expenditures:** The expenditures arise as salaries paid to staff and for other costs incurred in delivering care to its patrons. Furthermore Society provides facilities and other supplies, the cost of which must be recovered from operations.

Society policy requires that a capital renewal fund be established so that as the facility ages it can be kept in operating condition. Salaries are negotiated within the broader labour market and are thus subject to competition and associated uncertainties. Similarly supplies are purchased in markets subject to economic influences beyond the Society's control. Hence the estimated expenditures are uncertain.

g) **Terminal or Salvage Value:** Each project has a lifespan at the end of which there may be a terminal value resulting from the sale of an outdated facility or from converting it to another use. The terminal value is dependent on factors such as real estate markets and new or competing technologies to name but a few.

h) **The Discount Rate:** The purpose of discounting is to bring all cash flows to a common time period [i.e. the present] while accounting for the risks involved. Consequently the discount rate chosen must reflect the risks associated with the cash flows, the impact of inflation and alternative uses limited funds. *See also the section on Discount Rate – conceptual framework and Discount Rate – practical estimation approaches.*

Net Present Value Analysis – conceptual framework:
Net present value analysis is a widely accepted technique used in analyzing capital investment proposals. The essence of the approach is to compare the capital outlay [i.e. initial cost of the investment] to the economic value of the investment to the organization. The most common estimate of economic value is the present value of the future stream of net receipts expected [i.e. discounted future operating cash flows]. Present value derives from the notion that a dollar to be received a year from today is not as valuable as a dollar received today.

Net Present Value Analysis – decision rules:
The Net Present Value decision rule is simple, "Reject those projects with an anticipated negative net present value". Alternatively it states, "Accept those proposals promising a zero or a positive net present value". In effect the decision rule tells us to accept those projects that promise a greater economic value to the organization than they cost. Projects with negative net present values may sometimes be accepted to fulfill other goals of the organization, but these come at an economic cost.

Net Present Value Analysis – illustrative example:
The following example of a net present value situation is presented to facilitate an understanding of the method. The numbers chosen for the example are illustrative only; figures for an actual project will be considerably different and more detailed, but the methodology applies.

Suppose the Society is evaluating the economic consequences of expanding into a new community. A rigorous analysis has been done on the demographics of the community to determine the suitable size of facility.

Engineering consultants have advised that the facility will cost $9,602,394 to build and a builder is willing to give his guarantee to build it for this amount. We estimate that the project will have a life span of ten years at the end of which the Society expects to recover a salvage value of $1,000,000. This terminal value is not known with certainty.

After careful evaluation of the situation an expectation of annual revenue from the project of $3,500,000 is considered reasonable, as are the anticipated annual expenses of $2,000,000. Because these annual amounts are expected in the future they are not a certainty, in fact the net annual revenue may be more or less than the anticipated $1,500,000.

Because there is uncertainty in the estimated cash flows there is the risk of error. This risk can be allowed for by building a premium into the discount rate chosen to complete the analysis. Suppose that currently the prime lending rate is 5.00% and commercial mortgages are being written at 7.50%. Using the rule of prime plus twice the lenders premium establishes a discount rate of 10.00% [i.e. 5.00+2 x (7.50-5.00) = 10.00].

The first five columns of the following table summarize the problem while the last two demonstrate the approach.

A Net Present Value Analysis Illustration

Time Line	Annual Revenue	Annual Expenses	Salvage Value	Annual Cash Flow	PV Factor	Present Value
1	3,500,000	-2,000,000		1,500,000	0.9091	1,363,636
2	3,500,000	-2,000,000		1,500,000	0.8264	1,239,669
3	3,500,000	-2,000,000		1,500,000	0.7513	1,126,972
4	3,500,000	-2,000,000		1,500,000	0.6830	1,024,520
5	3,500,000	-2,000,000		1,500,000	0.6209	931,382
6	3,500,000	-2,000,000		1,500,000	0.5645	846,711
7	3,500,000	-2,000,000		1,500,000	0.5132	769,737
8	3,500,000	-2,000,000		1,500,000	0.4665	699,761
9	3,500,000	-2,000,000		1,500,000	0.4241	636,146
10	3,500,000	-2,000,000	1,000,000	2,500,000	0.3855	963,858
				Total present value (the economic value of the project)		9,602,394
				less capital outlay (cost of doing the project)		9,602,394
				equals net present value (NPV)		0

The column headed as “PV Factor” demonstrates the notion of present value. *(See also Table A.2 Present Value in the Appendix.)* A dollar promised in one year with money at 10% is worth only $0.91 [actually $0.9091]. This is equivalent to saying that $0.91 invested at 10% interest will grow to $1.00 in one year’s time. A dollar nine years away is worth only $0.42 [actually $0.4241]. Consequently the $1,500,000 in year six is worth only $846,711 in present value terms as shown in the column headed “Present Value”.

Summing the Present Value column tells us that the economic value of the project is $9,602,394 which is equal to the cost of building the project [i.e. the initial investment] hence the net present value is zero indicating that the Society would be justified in undertaking the project. Had the initial investment been less than the $9,602,394, say $9,500,000, the project would have a net present value of $102,394, thus being even more attractive. An outlay greater than $9,602,394, for example $10,000,000 would generate a negative net present value of -$397,606 signaling the rejection of the investment. The organization might still decide to go ahead to achieve a broader social goal within its Mission, but it would do so in full knowledge that it is subsidizing the project to the tune of $397,606.

Net Present Value Analysis – not-for-profit situations:
Is NPV analysis relevant for not-for-profit organizations? Even though NPV analysis has a very strong profit implication its underlying rigour offers great advantages in making wise investment decisions. In short it yields an economic benchmark against which alternatives can be measured; it adds another layer of information for sound decisions.

Net Present Value Analysis – process:
The net present value process involves three steps. First the cash flows must be estimated. Next the appropriate discount rate must be selected. Finally the net present value must be calculated. While the arithmetic of net present value analysis is fairly straight forward, as shown in the example, the gathering of information and assessing the risk implications require disciplined effort in preparing project proposals.

Present Value:
The present value is the discounted value of a future amount to be received at a stipulated discount rate a specified number of periods in the future. The relationship between future value and present value is calculated by the following formula.

$$PV = FV\left[\frac{1}{(1+i)^n}\right]$$

Here (PV) is the present value of (FV), a future value, (n) years away at arate of (i) per cent. *Please also see the section on Future Value. See also Table A.2 Present Value in the Appendix.*

Prices - nominal:
Nominal prices are those we observe in the market place at a particular point in time and in “the dollars of the day.” For example the price of regular gasoline at the pumps in 2002 was approximately 71.9¢ per litre. In 1960 one gallon of gasoline at the pumps cost approximately 18.9¢ or 4.14¢ per litre.

Prices – real:
Real prices are nominal prices from which the effects of inflation have been removed. Real prices are stated in the “dollars of a common time period.” The relationship between real prices and nominal prices is represented by the following formula.

$$P_t = P_k\left[\frac{CPI_t}{CPI_k}\right]$$

Here P_t is the (real) price in time *t* given the nominal price in time *k* (P_k) and CPI_t is the consumer price index from time *t*, and CPI_k is the consumer price index from time *k*.

For example the price of gasoline in 2002 in 2002 dollars was 71.9¢. In 1960 the price in 2002 dollars was 26.88¢ [i.e. 4.14x100/15.40=26.88]. While may be tempted to say that the price gasoline has increased by more than seventeen fold [nominal increase is 71.9/4.14=17.4] in reality the price has only tripled [real increase is 71.9/26.88=2.67 i.e. about three times]. Beware of comparing nominal prices, one to another, especially over long time periods. *(See also Inflation – historical rates.)*

Sinking Fund – definition:
A sinking fund is a fund set up to replace a wasting asset at the end of its useful life. Usually a regular annual sum is set aside to enable the fund, taking into account interest at the expected rate, to replace the exhausted asset at a specified date. Some have argued that amounts set aside for depreciation of an asset should be equal to the annual amounts needed to be placed in a notional sinking fund. *Source: Dictionary of Business, Oxford University Press, © Market House Books Ltd 1996.*

Sinking Fund – formula:
A sinking fund, i.e. the periodic amount needed to be set aside to accumulate a certain future lump sum can be calculated with the following formula.

$$A = FV\left[\frac{i}{(1+i)^n - 1}\right]$$

Here (A) is the annual payment needed at the end of each year to accumulate the desired future value, (FV) over (n) years at (i) per cent per annum. The sinking fund formula is the reciprocal of the future value of an annuity formula. *Please also see the Annuity – formulae section and Table A.4 Sinking Fund in the appendix.*

Table A.1 Amount at Compound Interest

Period	2.50%	5.00%	7.50%	10.00%	15.00%	20.00%	30.00%
1	1.0250	1.0500	1.0750	1.1000	1.1500	1.2000	1.3000
2	1.0506	1.1025	1.1556	1.2100	1.3225	1.4400	1.6900
3	1.0769	1.1576	1.2423	1.3310	1.5209	1.7280	2.1970
4	1.1038	1.2155	1.3355	1.4641	1.7490	2.0736	2.8561
5	1.1314	1.2763	1.4356	1.6105	2.0114	2.4883	3.7129
6	1.1597	1.3401	1.5433	1.7716	2.3131	2.9860	4.8268
7	1.1887	1.4071	1.6590	1.9487	2.6600	3.5832	6.2749
8	1.2184	1.4775	1.7835	2.1436	3.0590	4.2998	8.1573
9	1.2489	1.5513	1.9172	2.3579	3.5179	5.1598	10.6045
10	1.2801	1.6289	2.0610	2.5937	4.0456	6.1917	13.7858
11	1.3121	1.7103	2.2156	2.8531	4.6524	7.4301	17.9216
12	1.3449	1.7959	2.3818	3.1384	5.3503	8.9161	23.2981
13	1.3785	1.8856	2.5604	3.4523	6.1528	10.6993	30.2875
14	1.4130	1.9799	2.7524	3.7975	7.0757	12.8392	39.3738
15	1.4483	2.0789	2.9589	4.1772	8.1371	15.4070	51.1859
16	1.4845	2.1829	3.1808	4.5950	9.3576	18.4884	66.5417
17	1.5216	2.2920	3.4194	5.0545	10.7613	22.1861	86.5042
18	1.5597	2.4066	3.6758	5.5599	12.3755	26.6233	112.4554
19	1.5987	2.5270	3.9515	6.1159	14.2318	31.9480	146.1920
20	1.6386	2.6533	4.2479	6.7275	16.3665	38.3376	190.0496
21	1.6796	2.7860	4.5664	7.4002	18.8215	46.0051	247.0645
22	1.7216	2.9253	4.9089	8.1403	21.6447	55.2061	321.1839
23	1.7646	3.0715	5.2771	8.9543	24.8915	66.2474	417.5391
24	1.8087	3.2251	5.6729	9.8497	28.6252	79.4968	542.8008
25	1.8539	3.3864	6.0983	10.8347	32.9190	95.3962	705.6410
30	2.10	4.32	8.75	17.45	66.21	237.38	2,620
35	2.37	5.52	12.57	28.10	133.18	590.67	9,727
40	2.69	7.04	18.04	45.26	267.86	1,469.77	36,118
45	3.04	8.99	25.90	72.89	538.77	3,657.26	134,106
50	3.44	11.47	37.19	117.39	1,083.66	9,100.44	497,929

Note: This table gives the future value of $1.00 invested for ($n$) periods at ($i$) per cent, (e.g. $1.00 invested for 12 years at 7.50% accumulates to $2.38 i.e. from the factor 2.3818).

$$FV = PV(1+i)^n$$

Table A.2 Present Value

Period	2.50%	5.00%	7.50%	10.00%	15.00%	20.00%	30.00%
1	0.9756	0.9524	0.9302	0.9091	0.8696	0.8333	0.7692
2	0.9518	0.9070	0.8653	0.8264	0.7561	0.6944	0.5917
3	0.9286	0.8638	0.8050	0.7513	0.6575	0.5787	0.4552
4	0.9060	0.8227	0.7488	0.6830	0.5718	0.4823	0.3501
5	0.8839	0.7835	0.6966	0.6209	0.4972	0.4019	0.2693
6	0.8623	0.7462	0.6480	0.5645	0.4323	0.3349	0.2072
7	0.8413	0.7107	0.6028	0.5132	0.3759	0.2791	0.1594
8	0.8207	0.6768	0.5607	0.4665	0.3269	0.2326	0.1226
9	0.8007	0.6446	0.5216	0.4241	0.2843	0.1938	0.0943
10	0.7812	0.6139	0.4852	0.3855	0.2472	0.1615	0.0725
11	0.7621	0.5847	0.4513	0.3505	0.2149	0.1346	0.0558
12	0.7436	0.5568	0.4199	0.3186	0.1869	0.1122	0.0429
13	0.7254	0.5303	0.3906	0.2897	0.1625	0.0935	0.0330
14	0.7077	0.5051	0.3633	0.2633	0.1413	0.0779	0.0254
15	0.6905	0.4810	0.3380	0.2394	0.1229	0.0649	0.0195
16	0.6736	0.4581	0.3144	0.2176	0.1069	0.0541	0.0150
17	0.6572	0.4363	0.2925	0.1978	0.0929	0.0451	0.0116
18	0.6412	0.4155	0.2720	0.1799	0.0808	0.0376	0.0089
19	0.6255	0.3957	0.2531	0.1635	0.0703	0.0313	0.0068
20	0.6103	0.3769	0.2354	0.1486	0.0611	0.0261	0.0053
21	0.5954	0.3589	0.2190	0.1351	0.0531	0.0217	0.0040
22	0.5809	0.3418	0.2037	0.1228	0.0462	0.0181	0.0031
23	0.5667	0.3256	0.1895	0.1117	0.0402	0.0151	0.0024
24	0.5529	0.3101	0.1763	0.1015	0.0349	0.0126	0.0018
25	0.5394	0.2953	0.1640	0.0923	0.0304	0.0105	0.0014
30	0.47674	0.23138	0.11422	0.05731	0.01510	0.00421	0.00038
35	0.42137	0.18129	0.07956	0.03558	0.00751	0.00169	0.00010
40	0.37243	0.14205	0.05542	0.02209	0.00373	0.00068	0.00003
45	0.32917	0.11130	0.03860	0.01372	0.00186	0.00027	0.000007
50	0.29094	0.08720	0.02689	0.00852	0.00092	0.00011	0.000002

Note: This table gives the present value of $1.00 to be received ($n$) periods from today discounted at (i) per cent, (e.g. $1.00 to be received 10 years from today, discounted at 5% is worth $0.61 i.e. from the factor 0.6139).

$$PV = FV\left[\frac{1}{(1+i)^n}\right]$$

Table A.3 Amount of Annuity

Period	2.50%	5.00%	7.50%	10.00%	15.00%	20.00%	30.00%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0250	2.0500	2.0750	2.1000	2.1500	2.2000	2.3000
3	3.0756	3.1525	3.2306	3.3100	3.4725	3.6400	3.9900
4	4.1525	4.3101	4.4729	4.6410	4.9934	5.3680	6.1870
5	5.2563	5.5256	5.8084	6.1051	6.7424	7.4416	9.0431
6	6.3877	6.8019	7.2440	7.7156	8.7537	9.9299	12.7560
7	7.5474	8.1420	8.7873	9.4872	11.0668	12.9159	17.5828
8	8.7361	9.5491	10.4464	11.4359	13.7268	16.4991	23.8577
9	9.9545	11.0266	12.2298	13.5795	16.7858	20.7989	32.0150
10	11.2034	12.5779	14.1471	15.9374	20.3037	25.9587	42.6195
11	12.4835	14.2068	16.2081	18.5312	24.3493	32.1504	56.4053
12	13.7956	15.9171	18.4237	21.3843	29.0017	39.5805	74.3270
13	15.1404	17.7130	20.8055	24.5227	34.3519	48.4966	97.6250
14	16.5190	19.5986	23.3659	27.9750	40.5047	59.1959	127.9125
15	17.9319	21.5786	26.1184	31.7725	47.5804	72.0351	167.2863
16	19.3802	23.6575	29.0772	35.9497	55.7175	87.4421	218.4722
17	20.8647	25.8404	32.2580	40.5447	65.0751	105.9306	285.0139
18	22.3863	28.1324	35.6774	45.5992	75.8364	128.1167	371.5180
19	23.9460	30.5390	39.3532	51.1591	88.2118	154.7400	483.9734
20	25.5447	33.0660	43.3047	57.2750	102.4436	186.6880	630.1655
21	27.18	35.72	47.55	64.00	118.81	225.03	820.22
22	28.86	38.51	52.12	71.40	137.63	271.03	1,067.28
23	30.58	41.43	57.03	79.54	159.28	326.24	1,388.46
24	32.35	44.50	62.30	88.50	184.17	392.48	1,806.00
25	34.16	47.73	67.98	98.35	212.79	471.98	2,348.80
30	43.90	66.44	103.40	164.49	434.75	1,182	8,730
35	54.93	90.32	154.25	271.02	881.17	2,948	32,423
40	67.40	120.80	227.26	442.59	1,779.09	7,344	120,393
45	81.52	159.70	332.06	718.90	3,585.13	18,281	447,019
50	97.48	209.35	482.53	1,163.91	7,217.72	45,497	1,659,761

Note: This table gives the future value of $1.00 invested at the end of each of (n) periods at (i) per cent, (e.g. $1.00 to be invested at the end of each year for 10 years at 5% accumulates to $12.56 from the factor 12.5779).

$$FV = A\left[\frac{(1+i)^n - 1}{i}\right]$$

Table A.4 Sinking Fund

Period	2.50%	5.00%	7.50%	10.00%	15.00%	20.00%	30.00%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.4938	0.4878	0.4819	0.4762	0.4651	0.4545	0.4348
3	0.3251	0.3172	0.3095	0.3021	0.2880	0.2747	0.2506
4	0.2408	0.2320	0.2236	0.2155	0.2003	0.1863	0.1616
5	0.1902	0.1810	0.1722	0.1638	0.1483	0.1344	0.1106
6	0.1565	0.1470	0.1380	0.1296	0.1142	0.1007	0.0784
7	0.1325	0.1228	0.1138	0.1054	0.0904	0.0774	0.0569
8	0.1145	0.1047	0.0957	0.0874	0.0729	0.0606	0.0419
9	0.1005	0.0907	0.0818	0.0736	0.0596	0.0481	0.0312
10	0.0893	0.0795	0.0707	0.0627	0.0493	0.0385	0.0235
11	0.0801	0.0704	0.0617	0.0540	0.0411	0.0311	0.0177
12	0.0725	0.0628	0.0543	0.0468	0.0345	0.0253	0.0135
13	0.0660	0.0565	0.0481	0.0408	0.0291	0.0206	0.0102
14	0.0605	0.0510	0.0428	0.0357	0.0247	0.0169	0.0078
15	0.0558	0.0463	0.0383	0.0315	0.0210	0.0139	0.0060
16	0.0516	0.0423	0.0344	0.0278	0.0179	0.0114	0.0046
17	0.0479	0.0387	0.0310	0.0247	0.0154	0.0094	0.0035
18	0.0447	0.0355	0.0280	0.0219	0.0132	0.0078	0.0027
19	0.0418	0.0327	0.0254	0.0195	0.0113	0.0065	0.0021
20	0.0391	0.0302	0.0231	0.0175	0.0098	0.0054	0.0016
21	0.0368	0.0280	0.0210	0.0156	0.0084	0.0044	0.00122
22	0.0346	0.0260	0.0192	0.0140	0.0073	0.0037	0.00094
23	0.0327	0.0241	0.0175	0.0126	0.0063	0.0031	0.00072
24	0.0309	0.0225	0.0161	0.0113	0.0054	0.0025	0.00055
25	0.0293	0.0210	0.0147	0.0102	0.0047	0.0021	0.00043
30	0.0228	0.0151	0.0097	0.0061	0.00230	0.000846	0.000115
35	0.0182	0.0111	0.0065	0.0037	0.00113	0.000339	0.000031
40	0.0148	0.0083	0.0044	0.0023	0.00056	0.000136	0.000008
45	0.0123	0.0063	0.0030	0.0014	0.00028	0.000055	0.000002
50	0.0103	0.0048	0.0021	0.0009	0.00014	0.000022	0.000001

Note: This table gives the amount (A) to be set aside at the end of each of (n) periods at (i) per cent to accumulate $1.00, (e.g. to accumulate $1.00 in 6 years at 10.00% requires an annual investment of $0.13 at the end of each year i.e. from the factor 0.1296).

$$A = FV\left[\frac{i}{(1+i)^n - 1}\right]$$

Table A.5 Present Value of an Annuity

Period	2.50%	5.00%	7.50%	10.00%	15.00%	20.00%	30.00%
1	0.9756	0.9524	0.9302	0.9091	0.8696	0.8333	0.7692
2	1.9274	1.8594	1.7956	1.7355	1.6257	1.5278	1.3609
3	2.8560	2.7232	2.6005	2.4869	2.2832	2.1065	1.8161
4	3.7620	3.5460	3.3493	3.1699	2.8550	2.5887	2.1662
5	4.6458	4.3295	4.0459	3.7908	3.3522	2.9906	2.4356
6	5.5081	5.0757	4.6938	4.3553	3.7845	3.3255	2.6427
7	6.3494	5.7864	5.2966	4.8684	4.1604	3.6046	2.8021
8	7.1701	6.4632	5.8573	5.3349	4.4873	3.8372	2.9247
9	7.9709	7.1078	6.3789	5.7590	4.7716	4.0310	3.0190
10	8.7521	7.7217	6.8641	6.1446	5.0188	4.1925	3.0915
11	9.5142	8.3064	7.3154	6.4951	5.2337	4.3271	3.1473
12	10.2578	8.8633	7.7353	6.8137	5.4206	4.4392	3.1903
13	10.9832	9.3936	8.1258	7.1034	5.5831	4.5327	3.2233
14	11.6909	9.8986	8.4892	7.3667	5.7245	4.6106	3.2487
15	12.3814	10.3797	8.8271	7.6061	5.8474	4.6755	3.2682
16	13.0550	10.8378	9.1415	7.8237	5.9542	4.7296	3.2832
17	13.7122	11.2741	9.4340	8.0216	6.0472	4.7746	3.2948
18	14.3534	11.6896	9.7060	8.2014	6.1280	4.8122	3.3037
19	14.9789	12.0853	9.9591	8.3649	6.1982	4.8435	3.3105
20	15.5892	12.4622	10.1945	8.5136	6.2593	4.8696	3.3158
21	16.1845	12.8212	10.4135	8.6487	6.3125	4.8913	3.3198
22	16.7654	13.1630	10.6172	8.7715	6.3587	4.9094	3.3230
23	17.3321	13.4886	10.8067	8.8832	6.3988	4.9245	3.3254
24	17.8850	13.7986	10.9830	8.9847	6.4338	4.9371	3.3272
25	18.4244	14.0939	11.1469	9.0770	6.4641	4.9476	3.3286
30	20.9303	15.3725	11.8104	9.4269	6.5660	4.9789	3.332061
35	23.1452	16.3742	12.2725	9.6442	6.6166	4.9915	3.332991
40	25.1028	17.1591	12.5944	9.7791	6.6418	4.9966	3.333241
45	26.8330	17.7741	12.8186	9.8628	6.6543	4.9986	3.333308
50	28.3623	18.2559	12.9748	9.9148	6.6605	4.9995	3.333327

Note: This table gives the present value of a sum of $1.00 amounts ($A$) received at the end of each of (n) periods at (i) per cent, (e.g. $1.00 received for 12 years at 7.50% is worth $7.74 today i.e. from the factor 7.7353).

$$PV = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

Table A.6 Amortization Factors

Period	2.50%	5.00%	7.50%	10.00%	15.00%	20.00%	30.00%
1	1.0250	1.0500	1.0750	1.1000	1.1500	1.2000	1.3000
2	0.5188	0.5378	0.5569	0.5762	0.6151	0.6545	0.7348
3	0.3501	0.3672	0.3845	0.4021	0.4380	0.4747	0.5506
4	0.2658	0.2820	0.2986	0.3155	0.3503	0.3863	0.4616
5	0.2152	0.2310	0.2472	0.2638	0.2983	0.3344	0.4106
6	0.1815	0.1970	0.2130	0.2296	0.2642	0.3007	0.3784
7	0.1575	0.1728	0.1888	0.2054	0.2404	0.2774	0.3569
8	0.1395	0.1547	0.1707	0.1874	0.2229	0.2606	0.3419
9	0.1255	0.1407	0.1568	0.1736	0.2096	0.2481	0.3312
10	0.1143	0.1295	0.1457	0.1627	0.1993	0.2385	0.3235
11	0.1051	0.1204	0.1367	0.1540	0.1911	0.2311	0.3177
12	0.0975	0.1128	0.1293	0.1468	0.1845	0.2253	0.3135
13	0.0910	0.1065	0.1231	0.1408	0.1791	0.2206	0.3102
14	0.0855	0.1010	0.1178	0.1357	0.1747	0.2169	0.3078
15	0.0808	0.0963	0.1133	0.1315	0.1710	0.2139	0.3060
16	0.0766	0.0923	0.1094	0.1278	0.1679	0.2114	0.3046
17	0.0729	0.0887	0.1060	0.1247	0.1654	0.2094	0.3035
18	0.0697	0.0855	0.1030	0.1219	0.1632	0.2078	0.3027
19	0.0668	0.0827	0.1004	0.1195	0.1613	0.2065	0.3021
20	0.0641	0.0802	0.0981	0.1175	0.1598	0.2054	0.3016
21	0.0618	0.0780	0.0960	0.1156	0.1584	0.2044	0.3012
22	0.0596	0.0760	0.0942	0.1140	0.1573	0.2037	0.3009
23	0.0577	0.0741	0.0925	0.1126	0.1563	0.2031	0.3007
24	0.0559	0.0725	0.0911	0.1113	0.1554	0.2025	0.3006
25	0.0543	0.0710	0.0897	0.1102	0.1547	0.2021	0.3004
30	0.0478	0.0651	0.0847	0.1061	0.1523	0.20085	0.300115
35	0.0432	0.0611	0.0815	0.1037	0.1511	0.20034	0.300031
40	0.0398	0.0583	0.0794	0.1023	0.1506	0.20014	0.300008
45	0.0373	0.0563	0.0780	0.1014	0.1503	0.20005	0.300002
50	0.0353	0.0548	0.0771	0.1009	0.1501	0.20002	0.300001

Note: This table gives the amount (A) to be paid at the end of each of (n) periods at (i) per cent to discharge a debt of $1.00, (e.g. to discharge a debt of $1.00 in 6 years at an interest rate of 10.00% requires an annual payment of $0.23, i.e. from the factor 0.2296).

$$A = PV\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

About the Authors

Leonard Bauer

Len Bauer joined the University of Alberta in 1977 to assume research and teaching duties in agricultural business management, finance, and production economics. He was instrumental in creating the Agricultural Business Management Program at the University and was its first director.
He was guest professor at the University of Hohenheim in West Germany and guest lecturer at FINAFRICA in Milan, Italy, and at Curtin University of Technology in Perth, Australia. In 1995 he was workshop leader for agricultural economics instructors in Ukraine.
After retiring in 1996, Len continued developing instructional materials in Agricultural Business Management. Len, the Professor Emeritus of Agricultural Business Management in the Department of Rural Economy was dedicated to stamping out ignorance – wherever it was found. This set 'Agricultural Investment Analysis' was virtually complete prior to his death in 2004.

About the Collaborating Reviewers

Ted Darling

Ted Darling has been with Alberta Agriculture, Food and Rural Development since the mid-1970's, first as District Agriculturist and later as Farm Management Specialist. In 1990 he returned to the U of A for a Masters degree in Ag. Economics and he is currently an Agricultural Risk Specialist for the department based in Airdrie. Ted's interests lie in the area of individual firm management, and include risk, strategic planning, and innovative business arrangements.

Dean Dyck

Dean is the Financial Business Analyst - New Ventures with Alberta Agriculture, Food and Rural Development. Dean graduated with a Bachelor of Science in Agriculture in 1982 from the University of Saskatchewan with a major in Agricultural Economics. He has over 20 years of experience in farm business management, including positions as a Production Economist and Farm Management Agrologist with Saskatchewan Agriculture, and Farm Management Specialist with Alberta Agriculture. Dean's main interest is in financial, economic and risk analysis and production costs for new agricultural ventures.

Dale A. Kaliel

Dale's life is firmly rooted in agriculture. Hearkening from a mixed farm in northern Alberta, Dale received his B.Sc. Agric. (Animal Science) in 1977 followed by a M. Sc. Agric. (Ag. Econ.) in 1982 under the tutelage of Dr. Len Bauer. He has worked with

Alberta Agriculture, Food and Rural Development in a number of capacities since 1980 advancing to his current position of Sr. Economist: Production Economics.

The focus of Dale's career has revolved around creating economics, financial and business management information for Alberta producers and then striving to take them to the next, critical step ... showing them how to utilize their own "on-farm" information, applying fundamental economics principles and procedures, to make better business management decisions.

Frank Novak

Born and raised in Southern Alberta, Frank obtained his B. Sc. in Ag, and M. Sc. in Ag. Economics from the University of Alberta. Len Bauer was his advisor and mentor who urged him to continue to complete his Ph. D. in Ag. Econ. at the University of Illinois specializing in agricultural finance, farm management, and risk management. Frank taught at the U of A’s Department of Rural Economy from 1989 to 1999. He was a founding partner of Alberta Pig Company in 1995 and left the U of A to work full time in the industry in 1999. He is currently the managing director of Alberta Pig Company.

Brian Radke

Following graduation from the Western College of Veterinary Medicine in 1989, Brian practiced large animal medicine for 5 years, first in Ontario and then in BC’s Fraser Valley. He completed a Ph.D. in Agricultural Economics at Michigan State University in 1998. Brian is currently a Research Economist in the Economics and Competitiveness Division of Alberta Agriculture, Food and Rural Development, and previously held the position of Dairy Cattle Research Veterinarian with the Animal Industry Division. He is also an Adjunct Professor in the Department of Rural Economy, University of Alberta.