



# **A Growth and Yield Projection System (GYPSY) for Natural and Post-harvest Stands in Alberta**

## **Predicting Merchantable Density**

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## 1. Introduction

This report outlines the sub-models developed to estimate merchantable density for the four species groups modeled by GYPSY. These sub-models were designed to work in concert with the GYPSY sub-models published in May 2009 (Huang et al, 2009). For details on the other sub-models of GYPSY (i.e. top height, total density, basal area increment, and volume), see the May 21, 2009 publication.

## 2. Steps for Predicting Merchantable Stems/ha

1. Specify a utilization standard, e.g., 15/10 with a 0.2 m stump height;

2. Predict stump03 (i.e., stumpdob at 0.3 m stump ht):

$$\text{Rate} = a \cdot \exp[b \cdot (0.3 - \text{stumpht})^c] + 1$$

$$\text{Rate} = \text{stumpdob} / \text{stump03};$$

$$\text{Stumpdob} = \text{stumpdob at user-specified stumpht (stumpht} \leq 0.3 \text{ m)}$$

$$\text{Stump03} = \text{stumpdob at 0.3 m stumpht}$$

if stumpht=0.3 then do; stump03=stumpdob; end;

if stumpht<0.3 then do; stump03=stumpdob/rate; end;

3. Predict DBH:

$$\text{DBH} = (-k5 + \sqrt{k5^2 - 4 \cdot k6 \cdot (k4 - \text{stump03})}) / (2 \cdot k6);$$

[this is from the Stump03-DBH relationship, described in taper report #1]

4. Predict the CUM at the DBH, e.g.:

$$\text{CUM} = \frac{b_0}{1 + \exp[b_1 + b_2 \ln(\text{dbh} + 1) + b_3 \ln(\text{bhage} + 1) + b_4 \text{TopHT} + b_5 \ln(N + 1)]}$$

if CUM>100 then CUM=100;

5. Predict the merch stems:

$$\text{Merch\_stem} = \text{total density} \cdot (100 - \text{CUM}) / 100;$$

### 3. Diameter Prediction Models

Rate = stumpdob /stump03;

Stumpdob = stumpdob at user-specified stumpht (stumpht <= 0.3 m)

Stump03 = stumpdob at 0.3 m stumpht

$$\text{Rate} = a \cdot \exp[b \cdot (0.3 - \text{stumpht})^c] + 1$$

Parameter	AW	SB	SW	PL
A	0.009628	0.030241	0.01939	0.013081
B	10.82514	29.10004	14.58411	13.01001
C	1.068204	1.873577	1.286149	1.213286

if stumpht=0.3 then do; stump03=stumpdob; end;

if stumpht<0.3 then do; stump03=stumpdob/rate; end;

$$\text{DBH} = (-k5 + \sqrt{k5^2 - 4 \cdot k6 \cdot (k4 - \text{stump03})}) / (2 \cdot k6);$$

[this is from the Stump03-DBH relationship, described in taper report #1]

For example, for AW:

Obs	species	nregion	k4	k5	k6	stumpdob	stumpht	a	b	c	rate	stump03	DBH
1	AW	0	-0.33057	1.10664	.000986	5	0.1	0.009628	10.8251	1.06820	1.06700	4.6861	4.5150
2	AW	0	-0.33057	1.10664	.000986	5	0.2	0.009628	10.8251	1.06820	1.02429	4.8815	4.6902
3	AW	0	-0.33057	1.10664	.000986	5	0.3	0.009628	10.8251	1.06820	1.00963	5.0000	4.7964
4	AW	0	-0.33057	1.10664	.000986	10	0.1	0.009628	10.8251	1.06820	1.06700	9.3721	8.7003
5	AW	0	-0.33057	1.10664	.000986	10	0.2	0.009628	10.8251	1.06820	1.02429	9.7629	9.0479
6	AW	0	-0.33057	1.10664	.000986	10	0.3	0.009628	10.8251	1.06820	1.00963	10.0000	9.2587
7	AW	0	-0.33057	1.10664	.000986	15	0.1	0.009628	10.8251	1.06820	1.06700	14.0582	12.8550
8	AW	0	-0.33057	1.10664	.000986	15	0.2	0.009628	10.8251	1.06820	1.02429	14.6444	13.3726
9	AW	0	-0.33057	1.10664	.000986	15	0.3	0.009628	10.8251	1.06820	1.00963	15.0000	13.6864
10	AW	0	-0.33057	1.10664	.000986	20	0.1	0.009628	10.8251	1.06820	1.06700	18.7442	16.9798
11	AW	0	-0.33057	1.10664	.000986	20	0.2	0.009628	10.8251	1.06820	1.02429	19.5258	17.6649
12	AW	0	-0.33057	1.10664	.000986	20	0.3	0.009628	10.8251	1.06820	1.00963	20.0000	18.0802
13	AW	0	-0.33057	1.10664	.000986	25	0.1	0.009628	10.8251	1.06820	1.06700	23.4303	21.0754
14	AW	0	-0.33057	1.10664	.000986	25	0.2	0.009628	10.8251	1.06820	1.02429	24.4073	21.9257
15	AW	0	-0.33057	1.10664	.000986	25	0.3	0.009628	10.8251	1.06820	1.00963	25.0000	22.4409

## 4. Density Distribution Models

(use bold blue coefficients from mix-3 for this purpose)

Aspen and black spruce fit statistics

Parameter	Local		Population				
	AW	SB	AW	SB	AW	SB	SB
a0							<b>104.09</b>
b0	108.3000	113.2500	110.3810	115.6374	<b>110.37</b>	115.58	<b>3.8437</b>
b1	14.4012	6.5644	8.9488	3.3942	<b>8.9504</b>	3.3965	<b>0.5145</b>
b2	-6.1473	-4.7290	-4.2724	-3.1072	<b>-4.2732</b>	-3.1095	<b>-0.1358</b>
b3	1.0476	0.7098	1.3278	0.6031	<b>1.328</b>	0.6035	<b>0.2201</b>
b4	0.2584	0.2005	0.1255	0.1432	<b>0.1255</b>	0.1433	<b>-0.01063</b>
b5	-0.8537	-1.3708	-0.6409	-1.1954	<b>-0.641</b>	-1.1969	
c0							<b>0.5013</b>
c1							<b>0.0347</b>
c2							<b>0.2354</b>
$\sigma_{u_1}^2$	59.1992	30.1605					
$\sigma_{u_1u_2}$	-16.9030	-9.9025					
$\sigma_{u_2}^2$	4.9095	3.3228					
$\sigma^2$	17.6496	14.4417					
Model	[1]	[2a]	[1]	[2a]	<b>[1]</b>	[2a]	<b>[2b]</b>
Method	FOCE	FOCE	NLS	NLS	<b>Mix-3</b>	Mix-3	<b>Mix-3</b>

FOCE – first-order conditional expectation method

NLS – ordinary nonlinear least square method

Mix-3 – marginal structures method, based on FOCE

Distribution models:

[1]

$$\text{CUM} = \frac{b_0}{1 + \exp[(b_1 + u_{1i}) + (b_2 + u_{2i})\ln(\text{dbh} + 1) + b_3\ln(\text{bhage} + 1) + b_4\text{TopHT} + b_5\ln(N + 1)]}$$

[2a]

$$\text{CUM} = \frac{b_0}{1 + \exp[(b_1 + u_{1i}) + (b_2 + u_{2i})\ln(\text{dbh} + 1) + b_3\ln(\text{bhage} + 1) + b_4\text{TopHT} + b_5N/10000]}$$

[2b]

$$\text{CUM} = a_0 \left( 1 - \exp \left[ - \left( \frac{\text{DBH}}{b_0 (H_{\text{top}} - 1.3)^{b_1} N^{b_2} \text{Bhage}^{b_3} + b_4 \text{Bhage}} \right)^{c_0 \exp[c_1 (H_{\text{top}} - 1.3)] \text{Bhage}^{c_2}} \right] \right)$$

[3]

$$\text{CUM} = \frac{b_0}{1 + \exp[(b_1 + u_{1i}) + b_2\ln(\text{dbh} + 1) + b_3\ln(\text{bhage} + 1) + b_4\text{TopHT} + b_5N/10000]}$$

White spruce fit statistics

Parameter	Local		Population				
	Subregion		Subregion				
	1 to 3	Others	1 to 3	Others	All	All	All
a0							<b>102.99</b>
b0	112.2400	110.8600	119.0212	115.8741	115.7375	115.75	<b>4.0921</b>
b1	5.0735	5.3326	5.3555	3.0834	3.5627	3.5624	<b>0.6369</b>
b2	-3.5971	-3.5942	-2.5396	-2.3010	-2.3448	-2.3446	<b>-0.2063</b>
b3	0.6419	0.8063	0.1883	0.5661	0.5207	0.5207	<b>0.2555</b>
b4	0.1382	0.1088	0.0877	0.0757	0.0709	0.07087	<b>-0.01816</b>
b5	-6.3446	-5.8827	-6.7295	-4.6189	-5.0717	-5.072	
c0							<b>0.3233</b>
c1							<b>0.02614</b>
c2							<b>0.2457</b>
$\sigma_{u_1}^2$	20.7523	20.9305					
$\sigma_{u_1u_2}$	-6.0954	-5.9386					
$\sigma_{u_2}^2$	1.8238	1.7308					
$\sigma^2$	13.6108	16.9004					
Model	[2a]	[2a]	[2a]	[2a]	[2a]	[2a]	<b>[2b]</b>
Method	FOCE	FOCE	NLS	NLS	NLS	Mix-3	<b>Mix-3</b>

FOCE – first-order conditional expectation method

NLS – ordinary nonlinear least square method

Mix-3 – marginal structures method, based on FOCE



Lodgepole pine fit statistics

Parameter	Local		Population				
	Subregion		Subregion				
	Others	11	Others	11	All	All	All
a0							<b>100</b>
b0	104.2800	105.0000	100.0601	100.5066	100.7262	100.75	<b>14.4907</b>
b1	9.8465	10.7167	10.5075	10.5087	9.1578	9.1495	<b>0.7772</b>
b2	-6.2000	-6.5823	-5.3443	-5.6421	-5.2021	-5.1993	<b>-0.237</b>
b3	1.1060	0.6651	0.6654	0.3889	0.6966	0.6964	<b>-0.1114</b>
b4	0.2097	0.3211	0.1462	0.2526	0.1820	0.182	<b>0.03872</b>
b5	-4.9845	-7.7793	-5.6316	-8.1692	-5.3221	-5.3109	
c0							<b>1.5357</b>
c1							<b>0.02706</b>
c2							<b>0.07775</b>
$\sigma_{u_1}^2$	0.7939	1.0712					
$\sigma^2$	28.3778	28.3046					
Model	[3]	[3]	[3]	[3]	[3]	[3]	<b>[2b]</b>
Method	FOCE	FOCE	NLS	NLS	NLS	Mix-3	<b>Mix-3</b>

FOCE – first-order conditional expectation method

NLS – ordinary nonlinear least square method

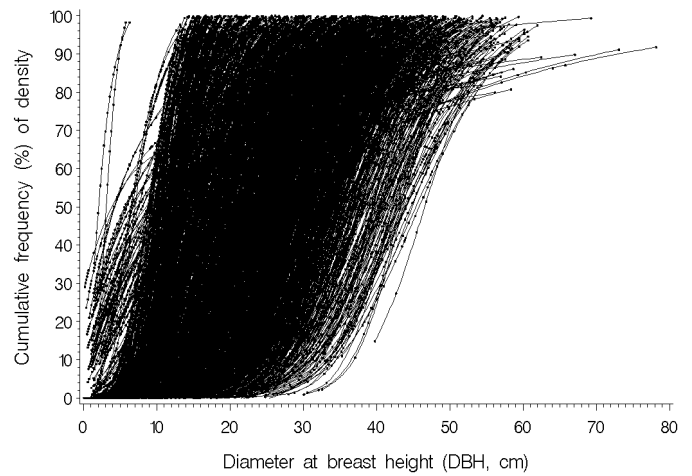
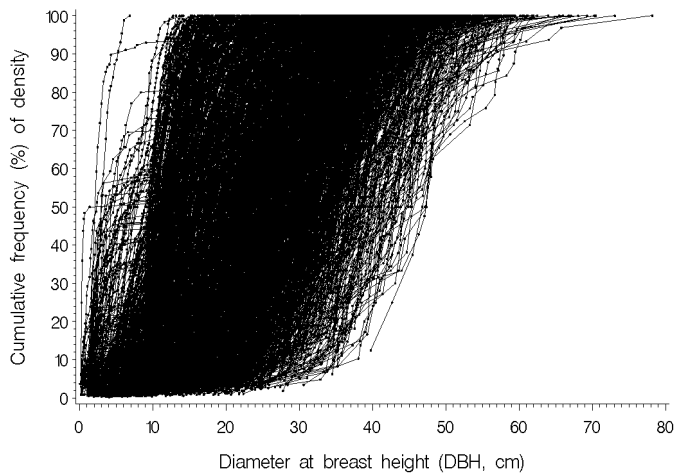
Mix-3 – marginal structures method, based on FOCE

## 5. Example localized curves from mixed models:

Aspen

$$\text{CUM} = \frac{b_0}{1 + \exp[(b_1 + u_{1i}) + (b_2 + u_{2i}) \ln(\text{dbh} + 1) + b_3 \ln(\text{bhage} + 1) + b_4 \text{TopHT} + b_5 \ln(N + 1)]}$$

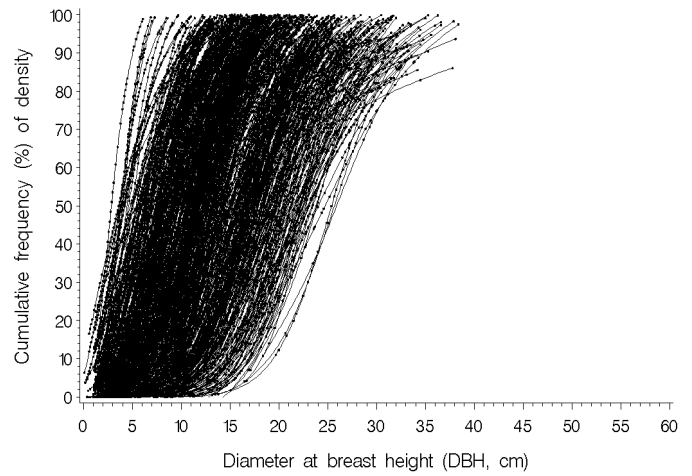
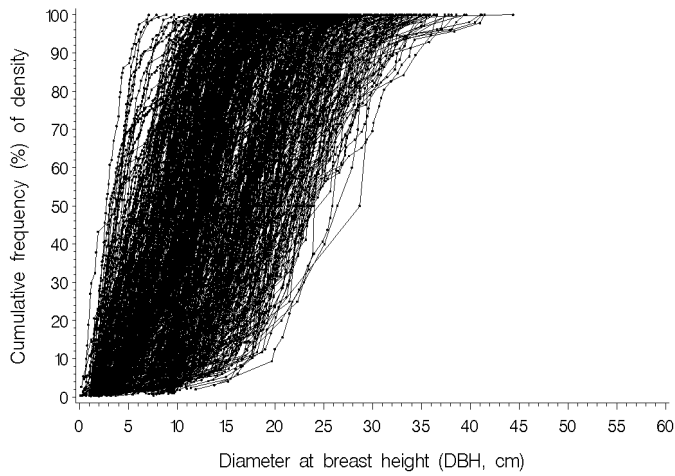
Aspen original data (left) vs fitted curves (right)



Black spruce

$$\text{CUM} = \frac{b_0}{1 + \exp[(b_1 + u_{1i}) + (b_2 + u_{2i}) \ln(\text{dbh} + 1) + b_3 \ln(\text{bhage} + 1) + b_4 \text{TopHT} + b_5 N/10000]}$$

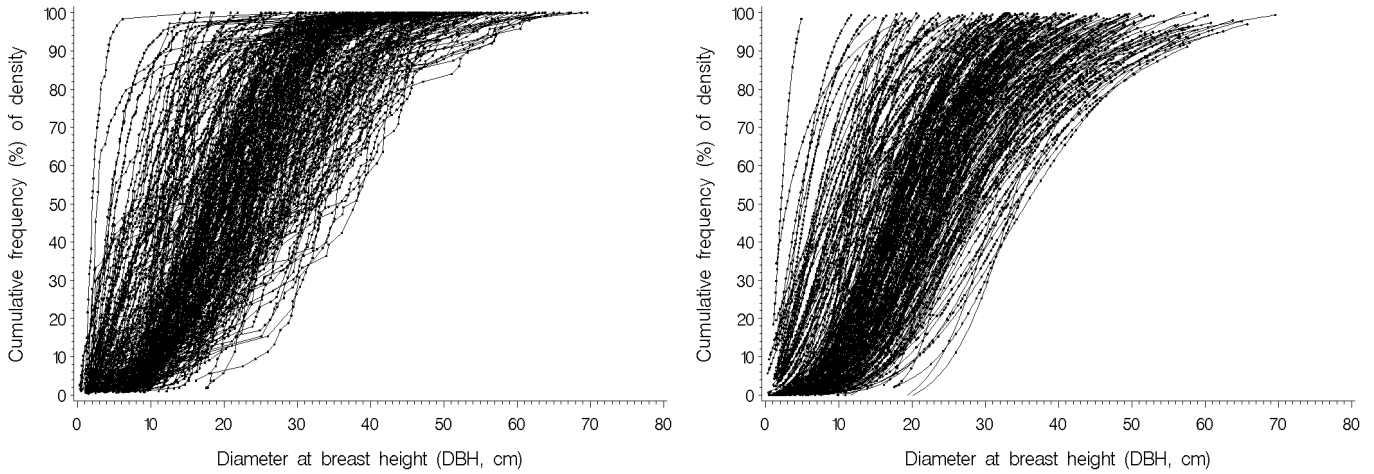
Black spruce original data (left) vs fitted curves (right)



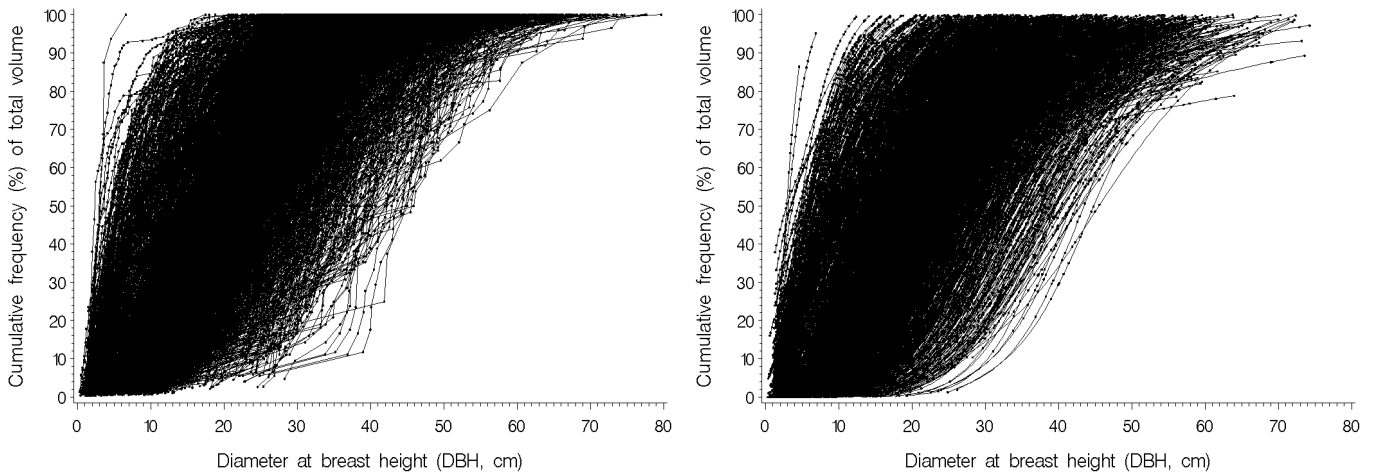
White spruce

$$\text{CUM} = \frac{b_0}{1 + \exp[(b_1 + u_{1i}) + (b_2 + u_{2i}) \ln(\text{dbh} + 1) + b_3 \ln(\text{bhage} + 1) + b_4 \text{TopHT} + b_5 N/10000]}$$

White spruce original data (left) vs fitted curves (right) – subregions 1 to 3



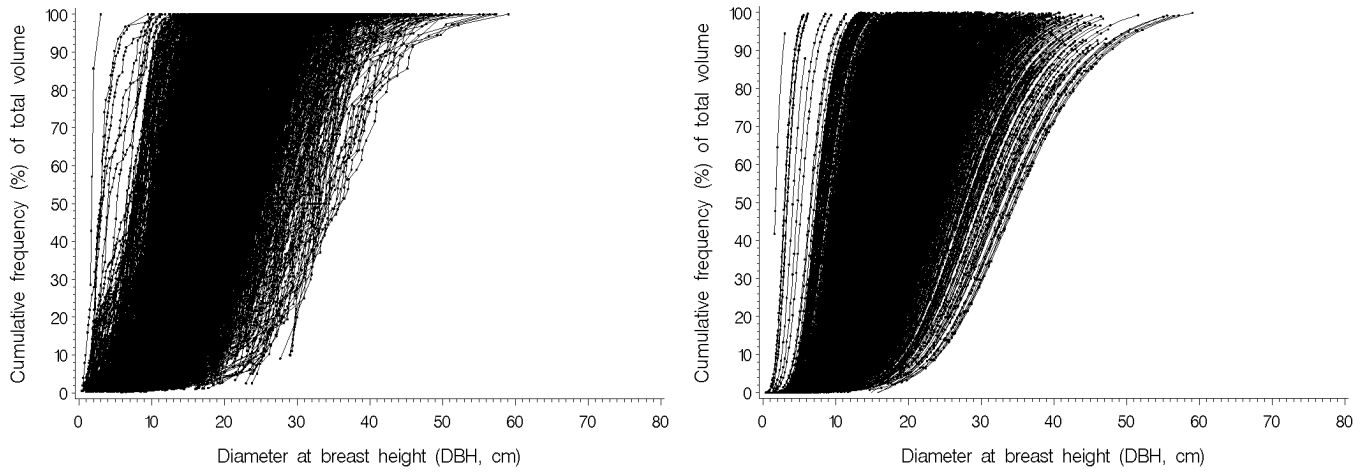
White spruce original data (left) vs fitted curves (right) – subregions 4 to 20 (y-axis should be density, not volume)



Lodgepole pine

$$\text{CUM} = \frac{b_0}{1 + \exp[(b_1 + u_{li}) + b_2 \ln(\text{dbh} + 1) + b_3 \ln(\text{bhage} + 1) + b_4 \text{TopHT} + b_5 N/10000]}$$

PL original data (left) vs fitted curves (right) – subregions 1 to 10, 12 to 20 (y-axis should be density, not volume)



PL original data (left) vs fitted curves (right) – subregion 11 (y-axis should be density, not volume)

