



A Growth and Yield Projection System (GYPSY) for Natural and Post-harvest Stands in Alberta

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1. Introduction

GYPSY (short for Growth and Yield Projection System) is a forest growth model composed of a number of sub-models or functions. Since the inception of the FRIAA-GYPSY project, several rounds of model development and testing have been completed. Fairly extensive validations were conducted by the model developers within Forest Management Branch as well as external consultants during the last six months. As a result of these validations and subsequent discussion, some of the GYPSY sub-models were re-formulated and re-fitted. This document lists the “final” sub-models effective May 21, 2009. Unless a substantial deficiency is detected with a particular sub-model during the application and use of GYPSY, this version of GYPSY will not be modified prior to March 2010.

Since most readers are already familiar with the sub-models, model structure and model usage, only the final sub-models and a brief description of each are provided here. A detailed technical document describing all sub-models is planned for release in March 2010. An Excel version of GYPSY along with a user’s manual is planned for release by January 2010. It is expected that many of the sub-models and/or the methodologies used to develop the sub-models will be published in scientific journals through anonymous peer-reviews.

2. Spatial Versus Non-Spatial GYPSY

The non-spatial version of GYPSY utilizes the following models:

- 1). Top height models (Section 4)
- 2). Non-spatial density models (Section 5)
- 3). Non-spatial basal area increment models (Section 6)
- 4). Gross total and merchantable volume models (Sections 10 and 11)

The spatial version of GYPSY utilizes the following models:

- 1). Top height models (Section 4)
- 2). Percent stocking models (Section 7)
- 3). Spatial density models (Section 8)
- 4). Spatial basal area increment models (Section 9)
- 5). Gross total and merchantable volume models (Sections 10 and 11)

In both cases, volume compilations use the nonlinear mixed-effects height-diameter models (Section 12) and the standard Alberta taper equations (1994).

Only the non-spatial version of GYPSY should be used for fire-origin stands.

Both the spatial and the non-spatial versions of GYPSY can be used for post-harvest stands. However, it is recommended that the spatial version of GYPSY be used whenever possible.

GYPSY was developed for four main Alberta tree species grown in pure and mixed-species stands: aspen (AW), lodgepole pine (PL), white spruce (SW), and black spruce (SB). All GYPSY sub-models are species-specific. In the cases where other species are present, they can in most cases be grouped with the four leading species as follows:

AW ← AW + PB

SB ← SB

SW ← SE + Fir

PL ← PJ + other pine species + LT

Based on the available data from Alberta government PSPs, grouping white birch (BW) with AW distorted the AW models substantially in most cases (due to the highly variable measurements for BW). Therefore, until more BW data are collected and more tests are done, it is recommended that BW should not be included in GYPSY runs. In the rare case where BW is managed as the leading species, BW could be considered as “AW” and its yields generated using the AW models, but the results obtained in this manner should be interpreted with caution.

3. Linking GYPSY to ARS Variables

One of the predominant reasons for developing GYPSY is to link the growth and yield of post-harvest stands to Alberta's Regeneration Performance Survey Standards and Alternative Regeneration Standards (ARS). In order to run the spatial version of GYPSY for post-harvest stands, the following variables must at a minimum be obtained for each cutblock or opening:

- 1). Top height
- 2). Total age
- 3). Density
- 4). Percent stocking

If alternative or enhanced forest management practices such as thinning, herbiciding, fertilization, and/or genetic improvement are applied to an opening, measurement of an additional variable "basal area" is recommended to enable the impacts of such practices to be assessed, and to achieve a better prediction accuracy:

- 5). Basal area

A brief description of each variable follows. Readers should consult Alberta's Regeneration Survey manual or relevant ARS manual for detailed field measurement procedures.

1). Top height

Top height is defined as the average height of the 100 largest DBH (diameter at 1.3 m above ground) trees per ha. For the regeneration survey grid used in Alberta, top height refers to the height of the largest (live) DBH tree in a 100 m² plot.

If the largest DBH tree in the 100 m² plot has a lost or broken top that has not yet been replaced with a new leader, the next largest diameter tree of that species is used as the top height tree. There are to be no substitution of lower diameter size trees due to health or other vigour conditions of the top height tree. Veterans, advanced growth, or obvious wolf trees must be excluded from top height trees. If necessary, relevant tree condition codes can be assigned to the top height trees (and other trees measured in the surveys), following the specifications given in the Regeneration Survey manual or ARS manual.

In the cases where all trees in a plot are shorter than 1.3 m (i.e., no DBH measurement), the top height tree is the largest diameter tree at stump height (0.3 m above ground).

2). Total age

Total age refers to the years since the point of germination. It is taken on the top height tree, regardless of whether the tree is shorter or taller than 1.3 m. Total age is used in conjunction with top height for determining site index.

Total age must be determined carefully, following the descriptions given in the Regeneration Survey manual or ARS manual.

3). Density

GYPHY uses different ways to define stand density for different stand types and species:

Post-harvest stands: for deciduous species, density refers to stems/ha of the subject species > 1.3 m tall; for coniferous species, density refers to stems/ha of the subject species > 0.3 m tall

Fire-origin stands: density refers to stems/ha of the subject species > 1.3 m tall, regardless of the coniferous or deciduous species.

4). Percent stocking

Percent stocking refers to the percentage (%) of the 10 m² plots of the regeneration survey grid with at least 1 tree of the subject species that meets the density count requirement. It is inferred from density measurements: no actual variable of “percent stocking” needs to be measured in the field.

5). Basal area

In stands where thinning, herbiciding, fertilization, genetic improvement, and/or other alternative or enhanced forest management practices are carried out, the diameters, along with the associated density, of some representative or all trees taller than 1.3 m need to be measured in some 10 m² plots on the regeneration survey grid, to obtain the “basal area at 1.3 m above ground” – a GYPHY input variable recommended when running projections for stands that have gone through these treatments:

- a). All diameter measurements are to be taken at breast height (1.3 m above ground). In the cases where the trees have not reached breast height, no DBH measurement is taken;
- b). The “1.3 density” (stems/ha for trees > 1.3 m tall) associated with the DBH measurements must be clearly recorded and separated from other regular variables.

Note that the DBH measurement and the “1.3 density” are additions to the regular variables for alternative or enhanced forest management stands. The “1.3 density” must be clearly separated from the regular density, which accounts for trees > 0.3 m tall for coniferous species. The recording of the data must allow for a clear separation of the 0.3 and 1.3 densities.

4. Top height models

Due to the variation in height growth patterns, different forms are used for the top height models for the four GYPSY species:

$$[1] \quad H_{top} = SI_t \times \left(\frac{1 + \exp(b_1 + b_2 \sqrt{\ln(1 + 50)} + b_3 [\ln(SI_t)]^2 + b_4 \sqrt{50})}{1 + \exp(b_1 + b_2 \sqrt{\ln(1 + totage)} + b_3 [\ln(SI_t)]^2 + b_4 \sqrt{50})} \right) \text{ (AW)}$$

$$[2] \quad H_{top} = SI_t \times \left(\frac{1 + \exp(b_1 + b_2 \sqrt{\ln(1 + 50)} + b_3 \ln(SI_t) + b_4 \sqrt{50})}{1 + \exp(b_1 + b_2 \sqrt{\ln(1 + totage)} + b_3 \ln(SI_t) + b_4 \sqrt{50})} \right) \text{ (SB and PL)}$$

$$[3] \quad H_{top} = SI_t \times \left(\frac{1 + \exp(b_1 + b_2 \sqrt{\ln(1 + 50^2)} + b_3 [\ln(SI_t)]^2 + b_4 \sqrt{50})}{1 + \exp(b_1 + b_2 \sqrt{\ln(1 + totage^2)} + b_3 [\ln(SI_t)]^2 + b_4 \sqrt{50})} \right) \text{ (SW)}$$

where:

H_{top} = top height (m), i.e., average height of the 100 largest DBH trees per ha
 SI_t = totage-based site index, i.e., top height at 50 years total age
 totage = total age from the point of germination

Estimated coefficients for the top height models are listed in Table 1.

Table 1. Estimated coefficients for the top height models.

Species	Aspen	Black spruce	Lodgepole pine	White spruce
Model	[1]	[2]	[2]	[3]
b_1	9.908888	14.56236	12.84571	12.14943
b_2	-3.92451	-6.04705	-5.73936	-3.77051
b_3	-0.32778	-1.53715	-0.91312	-0.28534
b_4	0.134376	0.240174	0.150668	0.165483

An example program is provided in Appendix 1 to demonstrate how the top height models can be used to predict site index and to make top height projections. Exact mathematical conversions between total age and breast height age (bhage), SI_t and SI_{bh} (SI_{bh} is the site index at 50 years bhage, which is currently used in Alberta), are embedded in the models (see Appendix 1). In the cases where stump age or breast height age needs to be converted to total age, the following average conversion factors can be used:

AW: total age = stump age + 0.5; total age = bhage + 4
 PL: total age = stump age + 3; total age = bhage + 8
 SW: total age = stump age + 4; total age = bhage + 12
 SB: total age = stump age + 5; total age = bhage + 15

5. Density models (non-spatial)

The density models describe stand density changes over time. They are functions of age, current or initial density, site quality and species mixtures. All models are species-specific. In GYPSY, stand density is defined differently for different stand types and species (see Section 3).

Aspen

$$[4] \quad N = SDF_{aw} \times \left(\frac{1 + \exp[b_1 + b_2 SI_{bh} + b_3 \ln(1 + 50)]}{1 + \exp[b_1 + b_2 SI_{bh} + b_3 \ln(1 + bhage)]} \right)$$

$$b_1 = - \left(\frac{1}{\sqrt{SDF_{aw} / 1000}} + \sqrt{1 + \frac{50}{\sqrt{SDF_{aw} \cdot \ln(1 + 50)}}}} \right) \times \ln(1 + 50)$$

$$b_2 = \frac{c_0}{4} \left(\sqrt{SDF_{aw}} \right)^{1/SDF_{aw}}$$

$$b_3 = (1 + c_0) SDF_{aw}^{\left[\frac{c_1 + \ln(SDF_{aw})}{SDF_{aw}} \right]}$$

$c_0 = 0.717966$, and $c_1 = 6.67468$.

Black spruce

$$[5] \quad N = SDF_{sb} \times \left(\frac{1 + \exp[b_1 + b_2 \ln(SI_{bh}) + b_3 \ln(1 + 50)]}{1 + \exp[b_1 + b_2 \ln(SI_{bh}) + b_3 \ln(1 + totage)]} \right)$$

$$b_1 = \frac{c_1}{\left(\sqrt{SDF_{sb} / 1000} + \ln(1 + 50) \right)^{c_2}}$$

$$b_2 = c_3$$

$$b_3 = c_3 SDF_{sb}^{1/SDF_{sb}}$$

$c_1 = -26.3836$, $c_2 = 0.166483$, and $c_3 = 2.738569$.

White spruce

$$[6] \quad N = SDF_{sw} \times \left(\frac{1 + \exp[b_1 + b_2 \ln(SI_{bh}) + b_3 \ln(1 + 50)]}{1 + \exp[b_1 + b_2 \ln(SI_{bh}) + b_3 \ln(1 + \text{totage})]} \right)$$

$$b_1 = \frac{c_1}{(\ln(SDF_{sw}) + \ln(1 + 50))^{c_2}} + z_1 \sqrt{1 + SDF_{aw} / 1000}$$

$$b_2 = c_3$$

$$b_3 = c_3 SDF_{sw}^{1/SDF_{sw}}$$

$$c_1 = -231.617, c_2 = 1.176995, \text{ and } c_3 = 1.733601.$$

if $SDF_{aw} = 0$ then $z_1 = 0$; if $SDF_{aw} > 0$ then $z_1 = 1$.

Lodgepole pine

$$[7] \quad N = SDF_{pl} \times \left(\frac{1 + \exp[b_1 + b_2 \ln(SI_{bh}) + b_3 \ln(1 + 50)]}{1 + \exp[b_1 + b_2 \ln(SI_{bh}) + b_3 \ln(1 + \text{totage})]} \right)$$

$$b_1 = \left(c_1 + \frac{z_1 (SDF_{aw} / 1000)}{2} + \frac{z_2 (SDF_{sw} / 1000)}{3} + \frac{z_3 (SDF_{sb} / 1000)}{4} \right) + \frac{c_2}{(\sqrt{SDF_{pl}})^{c_3}}$$

$$b_2 = \frac{c_4}{(\sqrt{SDF_{pl}})^{c_5}}$$

$$b_3 = c_4 SDF_{pl}^k, \quad k = \frac{1 + c_6 \sqrt{SDF_{pl}}}{SDF_{pl}}$$

$$c_1 = -5.25144, c_2 = -483.195, c_3 = 1.138167, c_4 = 1.017479, c_5 = -0.05471, \text{ and } c_6 = 4.11215.$$

if $SDF_{aw} = 0$ then $z_1 = 0$; if $SDF_{sw} = 0$ then $z_2 = 0$; if $SDF_{sb} = 0$ then $z_3 = 0$;
if $SDF_{aw} > 0$ then $z_1 = 1$; if $SDF_{sw} > 0$ then $z_2 = 1$; if $SDF_{sb} > 0$ then $z_3 = 1$.

Where in [4]-[7]:

N = density (stems/ha) of the subject species

SDF_{aw} = stand density factor for AW, which is N at 50 years bhage

SDF_{sb} = stand density factor for SB, which is N at 50 years totage

SDF_{sw} = stand density factor for SW, which is N at 50 years totage

SDF_{pl} = stand density factor for PL, which is N at 50 years totage

SI_{bh} = site index (m) of the subject species at 50 years bhage

6. Basal area increment models (non-spatial)

The basal area increment models provide annual basal area increment predictions. They are functions of breast height age, site index, density, basal area, and species composition (SC), where SC refers to the ratio of stand densities between the subject species and all species combined ($SC = N_{\text{species}}/N_{\text{all}}$). All models are species-specific. They can be used to make basal area predictions whether the current stand basal area is available or not.

Aspen

$$[8] \quad BAINC = \frac{10^{-4} a_1 B h a g e_1^2 \exp(-a_2 B h a g e_1^{(1/2+a_1)}) \cdot [\ln(1 + N_0 \sqrt{1 + B h a g e_1})]^2 S I_{aw} S C_1^{a_5}}{(1 + B A_1)^{a_3} \left[1 + \exp\left(1 - \frac{\ln(1 + S C_1^2)}{2}\right) \right]} + k$$

$$k = a_4 \ln(0.01 + B h a g e_1 / 10)$$

$$a_1 = 0.751313, a_2 = 0.018847, a_3 = 1.143762, a_4 = -0.03475, \text{ and } a_5 = 0.835189.$$

Black spruce

$$[9] \quad BAINC = 10^{-4} a_1 \exp(-a_2 B h a g e_1) B h a g e_1^{(\sqrt{a_1+a_2})} S C_1^{a_3} \cdot k$$

$$k = \frac{(1 + \sqrt{N_0} \cdot \sqrt{1 + B h a g e_1}) \ln(1 + S I_{sb}) \exp\left(-\frac{N_0 / 4}{10000}\right)}{(1 + B A_1)^{a_2}}$$

$$a_1 = 0.966285, a_2 = 0.056315, \text{ and } a_3 = 0.17191.$$

White spruce

$$[10] \quad BAINC = \frac{10^{-4} a_1 (a_2 + B h a g e_1)^2 (1 + B h a g e_1)^{(\sqrt{a_1+a_2-a_3})} \exp(-a_2 B h a g e_1) S C_1^{a_6} \cdot m}{\left[1 + \exp\left(1 + k + \frac{\ln(1 + \sqrt{N_0} / 10000)}{2} + a_3 \ln(1 + B A_1)\right) \right]}$$

$$m = [\ln(1 + N_0 \sqrt{1 + B h a g e_1})]^2 \sqrt{S I_{sw}} \exp\left(-\frac{N_0 / 10}{10000}\right)$$

$$k = a_4 z_1 \ln\left(1 + \frac{S D F_{aw}}{10000}\right) + a_5 z_2 \ln\left(1 + \frac{S D F_{pl}}{10000}\right) + z_3 \ln\left(1 + \frac{S D F_{sb}}{10000}\right)$$

$$a_1 = 0.089153, a_2 = 0.072171, a_3 = -0.11483, a_4 = 5.839408, \\ a_5 = 1.753002, a_6 = 0.239521.$$

if $S D F_{aw} = 0$ then $z_1 = 0$; if $S D F_{pl} = 0$ then $z_2 = 0$; if $S D F_{sb} = 0$ then $z_3 = 0$;
if $S D F_{aw} > 0$ then $z_1 = 1$; if $S D F_{pl} > 0$ then $z_2 = 1$; if $S D F_{sb} > 0$ then $z_3 = 1$.

Lodgepole pine

$$[11] \quad BAINC = \frac{10^{-4} a_1 Bhage_1 \exp(-a_2 Bhage_1) \left(1 + \frac{\ln(1 + Bhage_1)}{2}\right) \cdot m_1}{\left[1 + \exp\left(\frac{k}{2} + \ln\left(1 + \frac{1}{3} \cdot \frac{N_0}{10000}\right) - a_3 \sqrt{SC_1} + a_4 \ln(1 + BA_1)\right)\right]} + m_2$$

$$m_1 = (1 + a_3 + SI_{pl}^{a_6}) \sqrt{N_0} \exp\left(-\frac{1}{3} \cdot \frac{N_0}{10000}\right)$$

$$m_2 = a_5 \ln(0.01 + Bhage_1 / 10)$$

$$k = z_1 \ln\left(1 + \frac{SDF_{aw}}{1000}\right) + \frac{z_2}{2} \ln\left(1 + \frac{SDF_{sw}}{1000}\right) + \frac{z_3}{2} \ln\left(1 + \frac{SDF_{sb}}{1000}\right)$$

$$a_1 = 3.923984, a_2 = 0.05752, a_3 = 0.560402, a_4 = 0.672506, \\ a_5 = -0.00358, \text{ and } a_6 = 0.775765.$$

if $SDF_{aw} = 0$ then $z_1 = 0$; if $SDF_{sw} = 0$ then $z_2 = 0$; if $SDF_{sb} = 0$ then $z_3 = 0$;
if $SDF_{aw} > 0$ then $z_1 = 1$; if $SDF_{sw} > 0$ then $z_2 = 1$; if $SDF_{sb} > 0$ then $z_3 = 1$.

Where in [8]-[11]:

BAINC = annual basal area increment for the subject species (m²/ha/year)

Bhage₁ = current breast height age (years)

N₀ = initial density at bhage zero (for AW)

N₀ = initial density at totage zero (for SB, SW and PL)

SI_{species} = site index (SI_{bh}, m) at 50 years bhage

SC₁ = current species composition (N_{species}/N_{all})

BA₁ = current basal area (m²/ha) for the subject species

In practice, GYPSY users can assume that for aspen, the initial density at bhage zero is equivalent to the initial density at totage zero.

7. Percent stocking models

The percent stocking models describe the level of stocking and the clumpiness of trees in a stand. They provide a direct linkage between regeneration survey standards and yield forecasts. The models take the following forms:

$$[12] \quad PS_{aw} = PSI_{aw} \times \left(\frac{1 + \exp(b_1 + b_2[\ln(1 + 50)]^2 + b_3 \ln(SI_{aw}))}{1 + \exp(b_1 + b_2[\ln(1 + totage_{aw})]^2 + b_3 \ln(SI_{aw}))} \right)$$

$$[13] \quad PS_{sb} = PSI_{sb} \times \left(\frac{1 + \exp(b_1 + b_2[\ln(1 + 50)]^2 + b_3 \ln(SI_{sb}))}{1 + \exp(b_1 + b_2[\ln(1 + totage_{sb})]^2 + b_3 \ln(SI_{sb}))} \right)$$

$$[14] \quad PS_{sw} = PSI_{sw} \times \left(\frac{1 + \exp(b_1 + b_2[\ln(1 + 50)]^2 + b_3 \ln(SI_{sw}) + x_1(PSI_{aw}/b_3)/50)}{1 + \exp(b_1 + b_2[\ln(1 + totage_{sw})]^2 + b_3 \ln(SI_{sw}) + x_1(PSI_{aw}/b_3)/50)} \right)$$

$$[15] \quad PS_{pl} = PSI_{pl} \times \left(\frac{1 + \exp(b_1 + b_2[\ln(1 + 50)]^2 + b_3 \ln(SI_{pl}) + M)}{1 + \exp(b_1 + b_2[\ln(1 + totage_{pl})]^2 + b_3 \ln(SI_{pl}) + M)} \right),$$

$$M = b_4 x_1 PSI_{aw} + b_5 x_2 \sqrt{PSI_{sb}} + b_6 x_3 \sqrt{PSI_{sw}}$$

where:

PS = percent stocking (%), i.e., % of 10 m² plots with at least 1 tree of the subject species

PSI = percent stocking index, which is PS at 50 years total age

totage = total age (years) from the point of germination

SI = site index (m) at 50 years bhage

x₁, x₂ and x₃ are indicator variables defined as follows:

x₁ = 0 if PSI_{aw} = 0; x₂ = 0 if PSI_{sb} = 0; x₃ = 0 if PSI_{sw} = 0;

x₁ = 1 if PSI_{aw} > 0; x₂ = 1 if PSI_{sb} > 0; x₃ = 1 if PSI_{sw} > 0.

Estimated coefficients for the percent stocking models are listed in Table 2.

Table 2. Estimated coefficients for the percent stocking models.

Species	Aspen	Black spruce	White spruce	Lodgepole pine
Model	[12]	[13]	[14]	[15]
b ₁	-14.9555	-16.7154	-21.8721	-12.2731
b ₂	0.45162	0.36984	0.395767	0.23981
b ₃	1.59953	2.80854	4.128582	2.17610
b ₄				0.02492
b ₅				0.08990
b ₆				0.17058

8. Density models (spatial)

The spatial density models describe species-specific stand density changes over time as functions of age, current or initial density, site quality, species mixtures and percent stocking.

Aspen

$$[16] \quad N = SDF_{aw} \times \left(\frac{1 + \exp[b_1 + b_2 SI_{bh} + b_3 \ln(1 + 50)]}{1 + \exp[b_1 + b_2 SI_{bh} + b_3 \ln(1 + bhage)]} \right)$$

$$b_1 = - \left(\frac{1}{\sqrt{SDF_{aw}/1000}} + \sqrt{1 + \frac{50}{\sqrt{SDF_{aw}} \cdot \ln(1 + 50)}} + \frac{c_0}{c_1} \ln \left(1 + \frac{PSI_{aw}}{\sqrt{SDF_{aw}}} \right) \right) \times \ln(1 + 50)$$

$$b_2 = \frac{c_0}{4} \left(\sqrt{SDF_{aw}} \right)^{1/SDF_{aw}}$$

$$b_3 = (1 + c_0) SDF_{aw}^k$$

$$k = \frac{c_1 + \ln(SDF_{aw}) + \ln(1 + PSI_{aw}) / SDF_{aw}}{SDF_{aw}}$$

$$c_0 = 0.71123, \text{ and } c_1 = 7.015511.$$

Black spruce

$$[17] \quad N = SDF_{sb} \times \left(\frac{1 + \exp[b_1 + b_2 \ln(SI_{bh}) + b_3 \ln(1 + 50)]}{1 + \exp[b_1 + b_2 \ln(SI_{bh}) + b_3 \ln(1 + totage)]} \right)$$

$$b_1 = \frac{c_1}{\left(\sqrt{SDF_{sb}/1000} + \ln(1 + 50) \right)^{c_2}} - \frac{\ln(1 + PSI_{sb})}{1 + c_3}$$

$$b_2 = c_3$$

$$b_3 = c_3 SDF_{sb}^{1/SDF_{sb}}$$

$$c_1 = -25.7666, c_2 = 0.192351, \text{ and } c_3 = 2.718748.$$

White spruce

$$[18] \quad N = SDF_{sw} \times \left(\frac{1 + \exp[b_1 + b_2 \ln(SI_{bh}) + b_3 \ln(1 + 50)]}{1 + \exp[b_1 + b_2 \ln(SI_{bh}) + b_3 \ln(1 + totage)]} \right)$$

$$b_1 = \frac{c_1}{(\ln(SDF_{sw}) + \ln(1 + 50))^{c_2}} + z_1 \sqrt{1 + SDF_{aw}/1000} + \frac{\ln(1 + SDF_{sw})}{10}$$

$$b_2 = c_3$$

$$b_3 = c_3 \left(\sqrt{1 + PSI_{sw}} \right)^{-1/10} SDF_{sw}^{1/SDF_{sw}}$$

$$c_1 = -254.391, c_2 = 1.146709, \text{ and } c_3 = 2.202149.$$

if $SDF_{aw} = 0$ then $z_1 = 0$; if $SDF_{aw} > 0$ then $z_1 = 1$.

Lodgepole pine

$$[19] \quad N = SDF_{pl} \times \left(\frac{1 + \exp[b_1 + b_2 \ln(SI_{bh}) + b_3 \ln(1 + 50)]}{1 + \exp[b_1 + b_2 \ln(SI_{bh}) + b_3 \ln(1 + totage)]} \right)$$

$$b_1 = \left(c_1 + \frac{z_1 (SDF_{aw}/1000)}{2} + \frac{z_2 (SDF_{sw}/1000)}{3} + \frac{z_3 (SDF_{sb}/1000)}{4} \right) + m$$

$$m = \frac{c_2}{(\sqrt{SDF_{pl}})^{c_3}} - \frac{\ln(1 + PSI_{pl})}{1 + c_4}$$

$$b_2 = \frac{c_4}{(\sqrt{SDF_{pl}})^{c_5}}$$

$$b_3 = c_4 SDF_{pl}^k, \quad k = \frac{1 + c_6 \sqrt{SDF_{pl}}}{SDF_{pl}}$$

$$c_1 = -3.09711, c_2 = -512.394, c_3 = 1.14981, c_4 = 1.027108, c_5 = -0.05479, \text{ and } c_6 = 4.12191.$$

if $SDF_{aw} = 0$ then $z_1 = 0$; if $SDF_{sw} = 0$ then $z_2 = 0$; if $SDF_{sb} = 0$ then $z_3 = 0$;
if $SDF_{aw} > 0$ then $z_1 = 1$; if $SDF_{sw} > 0$ then $z_2 = 1$; if $SDF_{sb} > 0$ then $z_3 = 1$.

Where in [16]-[19]:

N = density (stems/ha) of the subject species

SDF_{aw} = stand density factor for AW, which is N at 50 years bhage

SDF_{sb} = stand density factor for SB, which is N at 50 years totage

SDF_{sw} = stand density factor for SW, which is N at 50 years totage

SDF_{pl} = stand density factor for PL, which is N at 50 years totage

SI_{bh} = site index (m) of the subject species at 50 years bhage

PSI = percent stocking index of the subject species (i.e., PS at 50 years totage)

9. Basal area increment models (spatial)

The spatial basal area increment models provide species-specific annual basal area increment predictions as functions of breast height age, site index, density, basal area, species composition (SC = $N_{\text{species}}/N_{\text{all}}$), and percent stocking. They can be used whether or not the current stand basal area is available.

Aspen

$$[20] \quad BAINC = \frac{10^{-4} a_1 Bhage_1^2 \exp(-a_2 Bhage_1) \cdot [\ln(1 + N_0 \sqrt{1 + Bhage_1})]^2 SI_{aw} SC_1^{a_5} PSI_{aw}^{a_6}}{(1 + BA_1)^{a_3} \left[1 + \exp\left(1 - \frac{\ln(1 + SC_1^2)}{2}\right) \right]} + k$$

$$k = a_4 \ln(0.01 + Bhage_1 / 10)$$

$$a_1 = 0.055906, a_2 = 0.054216, a_3 = 1.240035, a_4 = -0.05515, a_5 = 0.218326, \text{ and } a_6 = 0.749594.$$

Black spruce

$$[21] \quad BAINC = 10^{-4} a_1 \exp(-a_2 Bhage_1) Bhage_1^{(\sqrt{a_1 + a_2})} SC_1^{a_3} \cdot k$$

$$k = \frac{(1 + \sqrt{N_0} \cdot \sqrt{1 + Bhage_1}) \ln(1 + SI_{sb}) \exp\left(-\frac{N_0 / 4}{10000}\right) PSI_{sb}^{a_4}}{(1 + BA_1)^{a_2}}$$

$$a_1 = 0.333889, a_2 = 0.047669, a_3 = 0.03635, \text{ and } a_4 = 0.484915.$$

White spruce

$$[22] \quad BAINC = \frac{10^{-4} a_1 (a_2 + Bhage_1)^2 (1 + Bhage_1)^{(\sqrt{a_1 + a_2 - a_3})} \exp(-a_2 Bhage_1) SC_1^{a_6} \cdot m \cdot PSI_{sw}^{a_7}}{\left[1 + \exp\left(1 + k + \frac{\ln(1 + \sqrt{N_0} / 10000)}{2} + a_3 \ln(1 + BA_1)\right) \right]}$$

$$m = [\ln(1 + N_0 \sqrt{1 + Bhage_1})]^2 \sqrt{SI_{sw}} \exp\left(-\frac{N_0 / 10}{10000}\right)$$

$$k = a_4 z_1 \ln\left(1 + \frac{SDF_{aw}}{10000}\right) + a_5 z_2 \ln\left(1 + \frac{SDF_{pl}}{10000}\right) + z_3 \ln\left(1 + \frac{SDF_{sb}}{10000}\right)$$

$$a_1 = 0.041072, a_2 = 0.067012, a_3 = -0.00163, a_4 = 5.701986, a_5 = 2.222453, a_6 = 0.001126, a_7 = 0.368267.$$

if $SDF_{aw} = 0$ then $z_1 = 0$; if $SDF_{pl} = 0$ then $z_2 = 0$; if $SDF_{sb} = 0$ then $z_3 = 0$;
if $SDF_{aw} > 0$ then $z_1 = 1$; if $SDF_{pl} > 0$ then $z_2 = 1$; if $SDF_{sb} > 0$ then $z_3 = 1$.

Lodgepole pine

$$[23] \quad BAINC = \frac{10^{-4} a_1 Bhage_1 \exp(-a_2 Bhage_1) \left(1 + \frac{\ln(1 + Bhage_1)}{2}\right) \cdot m_1 \cdot PSI_{pl}}{\left[1 + \exp\left(\frac{k}{2} + \ln\left(1 + \frac{1}{3} \cdot \frac{N_0}{10000}\right) - a_3 \sqrt{SC_1} + a_4 \ln(1 + BA_1)\right)\right]} + m_2$$

$$m_1 = (1 + a_3 + SI_{pl}^{a_6}) \sqrt{N_0} \exp\left(-\frac{1}{3} \cdot \frac{N_0}{10000}\right)$$

$$m_2 = a_5 \ln(0.01 + Bhage_1 / 10)$$

$$k = z_1 \ln\left(1 + \frac{SDF_{aw}}{1000}\right) + \frac{z_2}{2} \ln\left(1 + \frac{SDF_{sw}}{1000}\right) + \frac{z_3}{2} \ln\left(1 + \frac{SDF_{sb}}{1000}\right)$$

$a_1 = 0.067391$, $a_2 = 0.054113$, $a_3 = 0.213767$, $a_4 = 0.977498$, $a_5 = -0.00452$, and $a_6 = 1.080421$.

if $SDF_{aw} = 0$ then $z_1 = 0$; if $SDF_{sw} = 0$ then $z_2 = 0$; if $SDF_{sb} = 0$ then $z_3 = 0$;
if $SDF_{aw} > 0$ then $z_1 = 1$; if $SDF_{sw} > 0$ then $z_2 = 1$; if $SDF_{sb} > 0$ then $z_3 = 1$.

Where in [20]-[23]:

BAINC = annual basal area increment for the subject species (m²/ha/year)

Bhage₁ = current breast height age (years)

N₀ = initial density at bhage zero (for AW)

N₀ = initial density at totage zero (for SB, SW and PL)

SI_{species} = site index (SI_{bh}, m) at 50 years bhage

SC₁ = current species composition (N_{species}/N_{all})

BA₁ = current basal area (m²/ha) for the subject species

PSI = species percent stocking index (i.e., percent stocking at 50 years totage)

In practice, GYPSY users can assume that for aspen, the initial density at bhage zero is equivalent to the initial density at totage zero.

10. Gross total volume models

The gross total volume models predict the species-specific gross total volume of a stand at a 0/0 utilization standard based on the top height and basal area of the subject species.

White spruce and black spruce:

$$[24] \quad Tvol = \beta_1 BA^{\beta_2} H_{top}^{\beta_3}$$

Aspen and lodgepole pine:

$$[25] \quad Tvol = \beta_1 BA^{\beta_2} H_{top}^{\beta_3} \exp\left(1 + \frac{\beta_4}{H_{top}^2 + 1}\right)$$

Where in [24]-[25]:

Tvol = gross total volume (m³/ha) of the subject species at the 0/0 utilization standard

BA = total basal area (m²/ha) of the subject species

H_{top} = top height (m) of the subject species

Estimated coefficients for the gross total volume models are listed in Table 3.

Parameter	Model [24]		Model [25]	
	Black spruce	White spruce	Aspen	Lodgepole pine
β_1	0.48628	0.41104	0.24872	0.19409
β_2	0.98296	0.98311	0.98568	0.98828
β_3	0.91060	0.97106	0.85728	0.94935
β_4			-24.9961	-3.39036

Given the observed basal area and top height of a stand from any inventory, the gross total volume models can be used independent of the other models to obtain accurate volume predictions.

11. Merchantable volume models

The merchantable volume models predict the species-specific merchantable volume of a stand at any user-defined utilization standard. They take the following general form:

$$[26] \quad Mvol = \frac{Tvol \cdot k^{b_0}}{b_1 H_{top}^{b_2} StumpDOB^{b_3} StumpHT^{b_4} TopDib^{b_5} k^{b_6} + k}$$

$$k = \sqrt{\frac{BA}{N/10000}}$$

where:

Mvol = merchantable volume (m³/ha) of the species at a specified utilization standard

Tvol = gross total volume (m³/ha) of the species at 0/0 utilization standard

BA = total basal area (m²/ha) of the subject species

H_{top} = species top height (m), i.e., average height of the 100 largest DBH trees per ha

StumpDOB = stump diameter outside bark (cm)

StumpHT = stump height (m)

TopDib = top diameter inside bark (cm)

N = density (stems/ha) of the species

Variables StumpDOB, StumpHT and TopDib define a utilization standard. For example, StumpDOB = 13, StumpHT = 0.3 and TopDib = 7 represent a 13/7 utilization standard with a stump height of 0.3 m. The minimum merchantable log length (e.g., 2.44, 3.66, or 4.88 m) variable had an insignificant impact on the merchantable volume models.

Estimated coefficients for the merchantable volume model [26] are listed in Table 4.

Table 4. Estimated coefficients for the merchantable volume model [26].

Parameter	Species			
	Aspen	Black spruce	White spruce	Lodgepole pine
b ₀	0.993673	0.98152	0.996262	0.989889
b ₁	923.5825	0.678011	7.021736	1.055091
b ₂	-3.96171	-1.10256	-1.77615	-0.19072
b ₃	3.366144	4.148139	1.91562	4.915593
b ₄	0.316236	0.511391	0.4111	0.42574
b ₅	0.968953	1.484988	1.024803	1.006379
b ₆	-1.61247	-3.26425	-0.80121	-4.87808

12. Height-diameter models

Height-diameter models are used to predict missing tree heights. They were developed using the first-order (FO) method of the nonlinear mixed-effects modeling technique. In addition to the four GYPSY species, height-diameter models were also developed for other tree species.

For each species, 3 separate models were fitted: a mixed model, a grand mean model, and a maximum model. The mixed model can be used to make accurate local predictions. The grand mean model is used to predict the population average. The maximum model can be used to constraint unrealistic local predictions (if they occur at all) from the mixed model approach, by increasing it by 10 or 20% and using the increased heights as the upper limits. For the four main species, subregion-based grand mean models were also fitted.

Mixed models (estimated parameters for the mixed models are listed in Tables 5a and 5b)

$$H_{ij} = 1.3 + (\beta_1 + b_{1i})[1 - \exp(-(\beta_2 + b_{2i})DBH_{ij})]^{(\beta_3 + b_{3i})} + \varepsilon_{ij} \text{ (AW, PJ, PB, BW)}$$

$$H_{ij} = 1.3 + (\beta_1 + b_{1i})[1 - \exp(-(\beta_2 + b_{2i})DBH_{ij})]^{\beta_3} + \varepsilon_{ij} \text{ (SB, PL)}$$

$$H_{ij} = 1.3 + (\beta_1 + b_{1i})[1 - \exp(-\beta_2 DBH_{ij})]^{(\beta_3 + b_{2i})} + \varepsilon_{ij} \text{ (FD)}$$

$$H_{ij} = 1.3 + \frac{(\beta_1 + b_{1i})}{1 + \exp[\beta_2 + (\beta_3 + b_{2i})\ln(DBH_{ij} + 1)]} + \varepsilon_{ij} \text{ (FB)}$$

$$H_{ij} = 1.3 + \frac{(\beta_1 + b_{1i})}{1 + \exp[(\beta_2 + b_{2i}) + (\beta_3 + b_{3i})\ln(DBH_{ij} + 1)]} + \varepsilon_{ij} \text{ (SW, LT)}$$

where:

H_{ij} , DBH_{ij} = tree height (m) and DBH (cm) for the j th tree in the i th plot

β_1 , β_2 and β_3 = fixed parameters common to all plots in the population

b_{1i} , b_{2i} , and b_{3i} = random parameters specific to plot i

ε_{ij} = a normally distributed within-plot error term

Table 5a. Estimated parameters for the mixed height-diameter models.

Parameter	Balsam fir	Douglas fir	Black spruce	Lodgepole pine
β_1	28.0508	18.8741	24.2436	22.8239
β_2	4.9970	0.07655	0.05014	0.05612
β_3	-1.7009	1.4970	1.0943	0.8931
$\sigma_{b_1}^2$	59.088	9.7027	61.0729	27.3425
$\sigma_{b_1 b_2}$	1.4997	1.1460	-0.1587	-0.07851
$\sigma_{b_2}^2$	0.04902	0.2049	0.000510	0.000469
σ^2	2.3383	1.7929	1.0361	1.2488

Table 5b. Estimated parameters for the mixed height-diameter models.

Parameter	White birch	Tamarack	Jack pine	Balsam poplar	Aspen	White spruce
β_1	23.8293	17.7012	16.8576	25.9183	23.1660	33.4845
β_2	0.05949	4.0050	0.1196	0.05309	0.09036	4.8067
β_3	1.0644	-1.5415	1.1992	1.0299	1.3587	-1.5657
$\sigma_{b_1}^2$	235.6300	39.4075	14.6007	26.3876	13.1749	51.3105
$\sigma_{b_1b_2}$	-0.9502	2.5992	-0.1057	-0.1313	-0.08264	0.8477
$\sigma_{b_2}^2$	0.004132	0.02595	0.001404	0.001080	0.001683	0.08428
$\sigma_{b_1b_3}$	-2.2586	0.3145	-0.3285	-0.8677	-0.2097	0.6358
$\sigma_{b_2b_3}$	0.01140	0.09888	0.004405	0.009951	0.01658	-0.02298
$\sigma_{b_3}^2$	0.04512	0.003591	0.02766	0.1164	0.2432	0.03203
σ^2	1.2051	0.9154	1.2668	2.7023	2.2005	2.8493

Note: $\sigma_{b_1}^2$, $\sigma_{b_2}^2$ and $\sigma_{b_3}^2$ are variances for random parameters b_1 , b_2 and b_3 , respectively, $\sigma_{b_1b_2}$, $\sigma_{b_1b_3}$ and $\sigma_{b_2b_3}$ are covariances between random parameters, and σ^2 is the error variance.

Maximum models (used to predict the maximum HT at a given DBH)

AW: $H = 1.3 + 36.2747[1 - \exp(-0.06984\text{DBH})]^{0.9526}$

SB: $H = 1.3 + 34.3451[1 - \exp(-0.04892\text{DBH})]^{0.9042}$

SW: $H = 1.3 + \frac{44.2305}{1 + \exp[4.3556 - 1.61271\ln(\text{DBH} + 1)]}$

PL: $H = 1.3 + 34.3351[1 - \exp(-0.09239\text{DBH})]^{1.2158}$

FB: $H = 1.3 + \frac{34.3627}{1 + \exp[4.3169 - 1.6948\ln(\text{DBH} + 1)]}$

BW: $H = 1.3 + 29.5987[1 - \exp(-0.07520\text{DBH})]^{1.0065}$

FD: $H = 1.3 + \frac{29.9011}{1 + \exp[3.8605 - 1.4021\ln(\text{DBH} + 1)]}$

LT: $H = 1.3 + \frac{24.8959}{1 + \exp[3.6910 - 1.78761\ln(\text{DBH} + 1)]}$

PJ: $H = 1.3 + 27.2158[1 - \exp(-0.09577\text{DBH})]^{1.3137}$

PB: $H = 1.3 + 37.3490[1 - \exp(-0.04811\text{DBH})]^{0.9098}$

Grand mean models (used to predict the population average HT at a given DBH)

AW: $H = 1.3 + 27.1624[1 - \exp(-0.06399DBH)]^{1.1293}$ (provincial)
 AW: $H = 1.3 + 27.9259[1 - \exp(-0.060671DBH)]^{1.2229}$ (subregions 4-11)
 AW: $H = 1.3 + 26.2387[1 - \exp(-0.07412DBH)]^{1.1985}$ (subregions 1-3, 12-20)

SB: $H = 1.3 + 26.3983[1 - \exp(-0.05419DBH)]^{1.2426}$ (provincial)
 SB: $H = 1.3 + 27.1949[1 - \exp(-0.05303DBH)]^{1.2372}$ (subregions 1-3, 12-20, 11)
 SB: $H = 1.3 + 23.4710[1 - \exp(-0.06309DBH)]^{1.3238}$ (subregions 4-10)

SW: $H = 1.3 + \frac{34.8284}{1 + \exp[5.3540 - 1.7532\ln(DBH + 1)]}$ (provincial)
 SW: $H = 1.3 + \frac{34.1575}{1 + \exp[5.7034 - 1.8929\ln(DBH + 1)]}$ (subregion 11-20)
 SW: $H = 1.3 + \frac{33.6474}{1 + \exp[4.7927 - 1.5373\ln(DBH + 1)]}$ (subregion 10)
 SW: $H = 1.3 + \frac{32.0551}{1 + \exp[4.9474 - 1.5706\ln(DBH + 1)]}$ (subregions 4-9)
 SW: $H = 1.3 + \frac{35.1067}{1 + \exp[5.2576 - 1.7589\ln(DBH + 1)]}$ (subregions 1-3)

PL: $H = 1.3 + 29.7809[1 - \exp(-0.05241DBH)]^{1.2389}$ (provincial)
 PL: $H = 1.3 + 29.1797[1 - \exp(-0.06138DBH)]^{1.3339}$ (subregion 11)
 PL: $H = 1.3 + 25.1275[1 - \exp(-0.07482DBH)]^{1.4598}$ (subregions 1-3, 10, 12-20)
 PL: $H = 1.3 + 19.2507[1 - \exp(-0.08169DBH)]^{1.2170}$ (subregions 4-9)

FB: $H = 1.3 + \frac{29.9784}{1 + \exp[5.2393 - 1.7470\ln(DBH + 1)]}$

BW: $H = 1.3 + 19.8793[1 - \exp(-0.09701DBH)]^{1.3233}$

FD: $H = 1.3 + \frac{29.9011}{1 + \exp[3.9017 - 1.2235\ln(DBH + 1)]}$

LT: $H = 1.3 + \frac{21.8411}{1 + \exp[5.4953 - 2.1154\ln(DBH + 1)]}$

PJ: $H = 1.3 + 23.5420[1 - \exp(-0.06962DBH)]^{1.2267}$

PB: $H = 1.3 + 29.6562[1 - \exp(-0.04166DBH)]^{0.9753}$

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Foothills Forest Products		✓
Hinton Wood Products	✓	✓
Millar Western Forest Products		✓
Sundance Forest Industries	✓	✓
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Appendix 1. A generalized program for predicting site index from top height and total age.

*This program predicts the following variables, from given top height and total age:

- SI_t (top height at 50 years totage)

- SI_{bh} (top height at 50 years bhage)

-Y2BH (years needed to reach the breast height of 1.3 m from germination)

```
OPTIONS LS=108 PS=60;
```

```
data examp1;
  input species $ topht totage;
  cards;
AW      20.00  50
AW      20.00  60
SB      20.00  50
SB      20.00  70
PL      20.00  50
PL      20.00  80
SW      20.00  50
SW      20.00  90
;
run;

data examp2;
  set examp1;

  if species='PL' then do;
      B1 = 12.84571;
      B2 = -5.73936;
      B3 = -0.91312;
      B4 = 0.150668;

      si0=10;
do until(abs(si0-si1)<0.00000001);
  k1 = exp(b1 + b2*sqrt(log(50+1.00)))+b3*(log(si0)**1+b4*sqrt(50));
  k2 = si0**(b3);
  k3 = ((si0)*(1+k1)/1.3-1)/(exp(b1)*exp(b4*sqrt(50))*k2);
  Y2BH_pl=exp((log(k3)/b2)**2) - 1;

  x10 = (1+exp(b1 + b2*sqrt(log(0 + totage + 1.00)) + b3*log(si0)**1+b4*sqrt(50)));
  x20 = (1+exp(b1 + b2*sqrt(log(50 + 1.00)) + b3*log(si0)**1+b4*sqrt(50)));
  si1=topht*x10/x20;
  si0=(si0+si1)/2;
end;

SI_t_pl = si1;
SI_bh_pl= si_t_pl*(1+exp(b1 + b2*sqrt(log(50 + 1.00))+b3*(log(si_t_pl)**1+b4*sqrt(50)))/
(1+exp(b1 + b2*sqrt(log(50 + Y2BH_pl + 1.00))+b3*(log(si_t_pl)**1+b4*sqrt(50))));
ht_pl = si_t_pl*(1+exp(b1 + b2*sqrt(log(50 + 1.00))+b3*(log(si_t_pl)**1+b4*sqrt(50)))/
(1+exp(b1 + b2*sqrt(log(totage + 1.00))+b3*(log(si_t_pl)**1+b4*sqrt(50))));
end;

if species='SB' then do;
      B1= 14.56236;
      B2= -6.04705;
      B3= -1.53715;
      B4= 0.240174;

      si0=10;
do until(abs(si0-si1)<0.00000001);
  k1 = exp(b1 + b2*sqrt(log(50+1.00)))+b3*(log(si0)**1+b4*sqrt(50));
  k2 = si0**(b3);
  k3 = ((si0)*(1+k1)/1.3-1)/(exp(b1)*exp(b4*sqrt(50))*k2);
  Y2BH_sb=exp((log(k3)/b2)**2) - 1;

  x10 = (1+exp(b1 + b2*sqrt(log(0 + totage + 1.00)) + b3*log(si0)**1+b4*sqrt(50)));
  x20 = (1+exp(b1 + b2*sqrt(log(50 + 1.00)) + b3*log(si0)**1+b4*sqrt(50)));
  si1=topht*x10/x20;
  si0=(si0+si1)/2;
end;

SI_t_sb = si1;
SI_bh_sb= si_t_sb*(1+exp(b1 + b2*sqrt(log(50 + 1.00))+b3*(log(si_t_sb)**1+b4*sqrt(50)))/
(1+exp(b1 + b2*sqrt(log(50 + Y2BH_sb + 1.00))+b3*(log(si_t_sb)**1+b4*sqrt(50))));
ht_sb = si_t_sb*(1+exp(b1 + b2*sqrt(log(50 + 1.00))+b3*(log(si_t_sb)**1+b4*sqrt(50)))/
(1+exp(b1 + b2*sqrt(log(totage + 1.00))+b3*(log(si_t_sb)**1+b4*sqrt(50))));
end;
```



```

if species='SW' then do;
    B1      =    12.14943;
    B2      =    -3.77051;
    B3      =    -0.28534;
    B4      =     0.165483;

    si0=10;
do until(abs(si0-sil)<0.00000001);
    k1 = exp(b1 + b2*sqrt(log(50**2+1.00))+b3*(log(si0))**2+b4*sqrt(50));
    k2 = si0**(b3*log(si0));
    k3 = ((si0)*(1+k1)/1.3-1)/(exp(b1)*exp(b4*sqrt(50))*k2);
    Y2BH_sw=sqrt(exp((log(k3)/b2)**2) - 1);

    x10 = (1+exp(b1 +b2*sqrt(log(0 +totage**2 +1.00)) +b3*log(si0)**2+b4*sqrt(50)));
    x20 = (1+exp(b1 +b2*sqrt(log(50**2 +1.00)) +b3*log(si0)**2+b4*sqrt(50)));
    sil=topht*x10/x20;
    si0=(si0+sil)/2;
end;

SI_t_sw = sil;
SI_bh_sw= si_t_sw*(1+exp(b1 + b2*sqrt(log(50**2 +1.00))+b3*(log(si_t_sw))**2+b4*sqrt(50)))/
(1+exp(b1 + b2*sqrt(log((50 +Y2BH_sw)**2 +1.00))+b3*(log(si_t_sw))**2+b4*sqrt(50)));
ht_sw = si_t_sw*(1+exp(b1 + b2*sqrt(log(50**2 +1.00))+b3*(log(si_t_sw))**2+b4*sqrt(50)))/
(1+exp(b1 + b2*sqrt(log((totage )**2 +1.00))+b3*(log(si_t_sw))**2+b4*sqrt(50)));
end;

if species='AW' then do;
    B1      =     9.908888;
    B2      =    -3.92451;
    B3      =    -0.32778;
    B4      =     0.134376;

    si0=10;
do until(abs(si0-sil)<0.00000001);
    k1 = exp(b1 + b2*sqrt(log(50+1.00))+b3*(log(si0))**2+b4*sqrt(50));
    k2 = si0**(b3*log(si0));
    k3 = ((si0)*(1+k1)/1.3-1)/(exp(b1)*exp(b4*sqrt(50))*k2);
    Y2BH_aw=exp((log(k3)/b2)**2) - 1;

    x10 = (1+exp(b1 +b2*sqrt(log(0 +totage +1.00)) +b3*log(si0)**2+b4*sqrt(50)));
    x20 = (1+exp(b1 +b2*sqrt(log(50 +1.00)) +b3*log(si0)**2+b4*sqrt(50)));
    sil=topht*x10/x20;
    si0=(si0+sil)/2;
end;

SI_t_aw = sil;
SI_bh_aw= si_t_aw*(1+exp(b1 + b2*sqrt(log(50 +1.00))+b3*(log(si_t_aw))**2+b4*sqrt(50)))/
(1+exp(b1 + b2*sqrt(log(50 +Y2BH_aw +1.00))+b3*(log(si_t_aw))**2+b4*sqrt(50)));
ht_aw = si_t_aw*(1+exp(b1 + b2*sqrt(log(50 +1.00))+b3*(log(si_t_aw))**2+b4*sqrt(50)))/
(1+exp(b1 + b2*sqrt(log(totage +1.00))+b3*(log(si_t_aw))**2+b4*sqrt(50)));
end;

drop B1      B2      B3      B4      si0      sil      k1      k2      k3 x10 x20;
proc print data=examp2(obs=10);
run;

*Output;

Obs  species  topht  totage  Y2BH_pl  SI_t_pl  SI_bh_pl  ht_pl  Y2BH_sb  SI_t_sb  SI_bh_sb
1     AW      20     50
2     AW      20     60
3     SB      20     50
4     SB      20     70
5     PL      20     50     5.59552  20.0000  21.3314  20.0000
6     PL      20     80     7.11354  15.0905  16.5410  20.0000
7     SW      20     50
8     SW      20     90

Obs  ht_sb  Y2BH_sw  SI_t_sw  SI_bh_sw  ht_sw  Y2BH_aw  SI_t_aw  SI_bh_aw  ht_aw
1
2
3  20.0000
4  20.0000
5
6
7  6.27265  20.0000  21.7585  20.0000
8  9.54306  12.2179  14.3948  20.0000

```