Ecologically Based Individual Tree Volume Estimation for Major Alberta Tree Species

Report # 1

Individual Tree Volume Estimation Procedures for Alberta:

Methods of Formulation and Statistical Foundations

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REPORT SERIES:

ECOLOGICALLY BASED INDIVIDUAL TREE VOLUME ESTIMATION FOR MAJOR ALBERTA TREE SPECIES

PUBLICATIONS FOR THIS REPORT SERIES INCLUDE THE FOLLOWING:

Report 1.	Individual tree volume estimation procedures for Alberta: methods of formulation and statistical foundations
Report 2.	Ecologically based individual tree height-diameter models for major Alberta tree species
Report 3.	Summary of equations and estimated coefficients for ecologically based individual tree volume estimation in Alberta
Report 4.	Ecologically based individual tree volume tables for balsam fir (*Abies balsamea* (L.) Mill.)
Report 5.	Ecologically based individual tree volume tables for white spruce (*Picea glauca* (Moench) Voss)
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Report 9.	Ecologically based individual tree volume tables for aspen (*Populus tremuloides* Michx.)
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Report 11.	Provincial-based individual tree volume tables for: 1). Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) 2). White birch (*Betula papyrifera* Marsh.) 3). Tamarack (*Larix laricina* (Du Roi) K. Koch) 4). Engelmann spruce (*Picea engelmannii* Parry ex Engelm.) 5). Jack pine (*Pinus banksiana* Lamb.)
Report 12.	Ecologically based individual tree volume tables for hardwood groups

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ABSTRACT

This report describes in detail the methods for predicting individual tree gross total volume, gross merchantable volume to any specified top diameter inside bark, merchantable length, and number of trees per cubic metre of merchantable volume (trees/m^3) for major Alberta tree species. The report also lists models and estimated natural region based coefficients associated with volume and other individual tree-related variable estimations, along with appropriate instructions for using them. Other publications of this series, Ecologically Based Individual Tree Volume Estimation for Major Alberta Tree Species, display tables that were formulated according to procedures described herein. Advantages of these ecologically based individual tree volume estimations include more accurate volume predictions from the taper equation, an integrated use of individual tree relationships, use of the newly classified Natural Regions instead of the Volume Sampling Regions, and a new layout form of the tables to facilitate practical use.

1.0 INTRODUCTION

This report describes in detail the methods of formulation and statistical foundations for ecologically based, individual tree volume estimation in Alberta. It provides the bases for construction of the tables displayed in other reports of this series, and includes procedures for predicting individual tree gross total volume, gross merchantable volume to any specified top diameter inside bark, merchantable length, and number of trees per cubic metre of merchantable volume (trees/m^3) for major Alberta tree species. All models and estimated natural region based coefficients associated with volume and other variable estimations related to individual trees are listed in this report, along with appropriate instructions and procedures for using them. Natural Regions of Alberta are those defined Appendix 4 and shown in Figure 1.

Compared to the previously used volume estimation methods adopted by Alberta Land and Forest Services (Alberta Forest Service 1985*a*), ecologically based individual tree volume estimation has several distinct features including the following:

1. A taper equation is used to predict individual tree gross total volume, gross merchantable volume, merchantable length, and number of trees per cubic metre of merchantable volume (trees/m^3). Taper equations have been shown to provide more accurate volume predictions for major Alberta tree species (LeMay 1982). More recently, the use of taper equations for individual tree volume estimation has become an increasingly popular trend (Flewelling 1993; Flewelling and Raynes 1993; Kozak 1988, 1991; Newnham 1992; Perez et al. 1990).

2. The relationships between tree height and tree diameter, diameter outside bark and inside bark, stump diameter and breast height diameter are used in an integrated manner. This approach greatly facilitates volume estimation.

3. Natural regions are used instead of the Volume Sampling Regions (VSRs, see Alberta Forest Service 1985*b*). Predictive relationships and volume tables were built on recently classified natural regions to reflect a refinement over previously defined VSR boundaries, and to emphasize the ever-increasing importance of ecology-based forest management in Alberta.

4. A new layout form of the volume tables was developed to facilitate practical use.

An example Statistical Analysis System (SAS) program showing the step-by-step computations for merchantable length, gross merchantable volume, gross total volume, trees/m^3 merchantable volume, total tree height, and stump diameter is attached (Appendix 3). Additional programs used for other computations, or for creating tables or matrices of user-defined ranges, are available upon request.

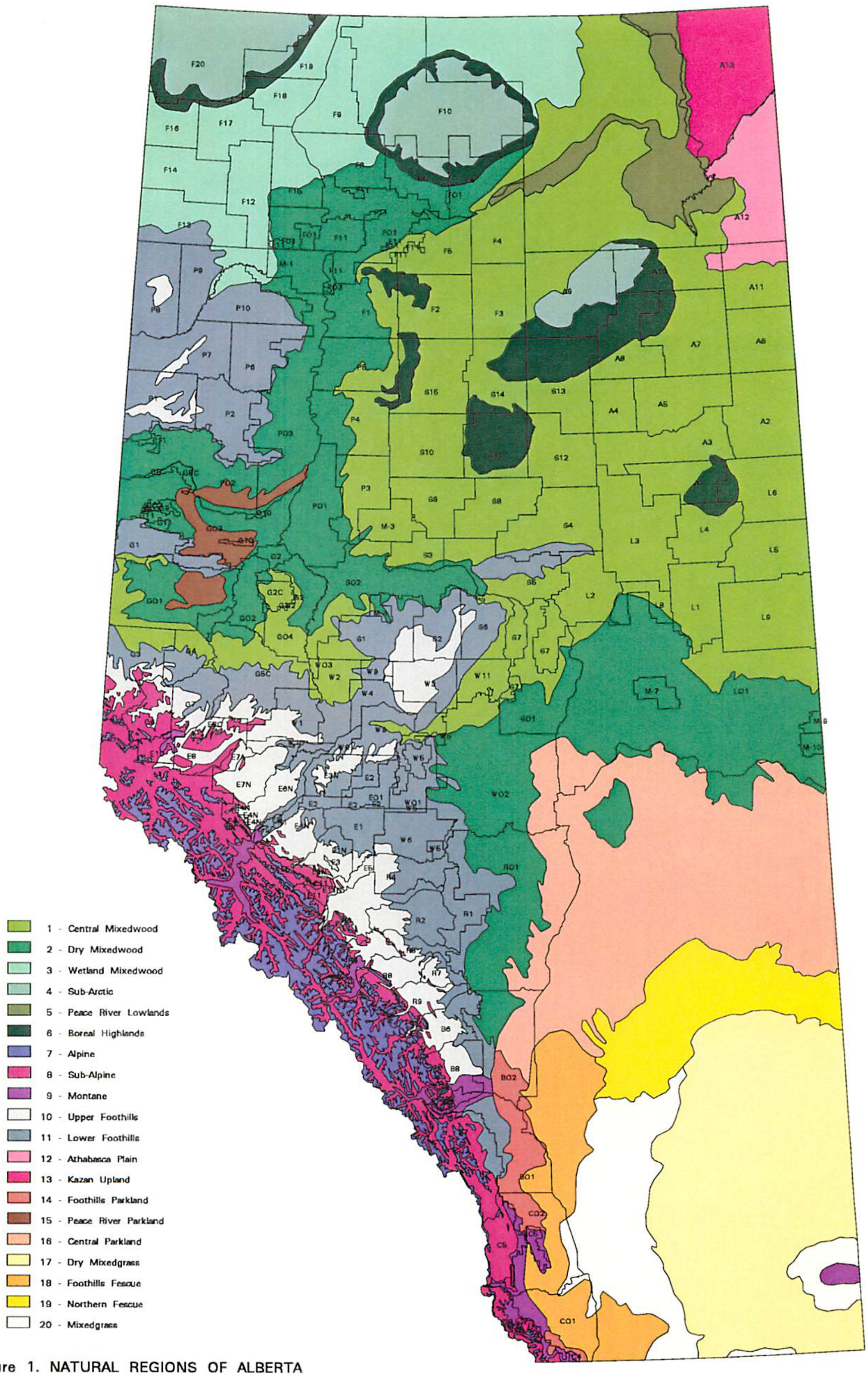

Figure 1. NATURAL REGIONS OF ALBERTA

Produced by Forest Resource Information Branch, Edmonton

Scale 1:5 000 000

2.0 THE BASE MODELS

The base models used to creat volume tables include the taper model, the diameter outside/inside bark model, the height-diameter model, and the stump diameter and breast height diameter model. All models are fitted on the provincial tree sectioning data (Alberta Forest Service 1988, 1993).

2.1 The Taper Model

Evaluation of alternative taper equations suggested that the variable-exponent taper equation (Kozak 1988) was appropriate for major Alberta tree species:

[1] $$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

where

[2] $$X = (1-\sqrt{h/H}) / (1-\sqrt{p})$$

and

d = diameter inside bark (cm) at h

h = height above the ground (m), $0 \leq h \leq H$

H = total tree height (m)

D = diameter at breast height outside bark (cm)

$Z = h / H$

p = location of the inflection point, assumed to be at 22.5% of total height above the ground

e = base of the natural logarithm ($\approx$2.71828)

a_0, a_1, a_2, b_1, b_2, b_3, b_4, b_5 = parameters to be estimated.

The location of the inflection point has been found to have little effect on the predictive properties of the taper model (Perez et al. 1990). A constant p value of 0.225 (the midpoint of the p ranges suggested by Kozak) was found appropriate for all Alberta tree species. Fitting of the taper equation requires nonlinear least squares procedures, with initial values of the parameters estimated from the linearized equation (see Appendix 1 for details). Results of the estimated natural region based coefficients for the taper model are listed in Appendix 2. Use of the model will be discussed in subsequent sections.

2.2 The Diameter Outside/Inside Bark Model

A linear equation expressing diameter outside bark as a function of diameter inside bark was fitted on the provincial stem analysis data

$$[3] \qquad DOB = a + bDIB$$

where:

DOB = diameter outside bark (cm) at any point on the stem

DIB = corresponding diameter inside bark (cm)

a, b = parameters to be estimated.

A relationship between diameters outside and inside bark is commonly used for conversion between these two diameters. The relationship can be used to predict diameter outside bark from diameter inside bark, or to predict diameter inside bark from diameter outside bark. For the latter, equation [3] is rearranged as follows:

$$[4] \qquad DIB = \frac{DOB-a}{b}$$

The relationship between diameter outside and diameter inside bark can also be used to compute bark thickness and, in conjunction with other equations, bark volume. Results of the estimated coefficients by natural regions for the diameter outside/inside bark model are listed in Appendix 2.

2.3 The Height-Diameter Model

A Richards-type height-diameter model (Huang et al. 1992) was found appropriate for major

Alberta tree species:

$$[5] \qquad H = 1.3 + a(1 - e^{-bD})^{c}$$

where:

H = total tree height (m)

D = diameter at breast height outside bark (cm)

e = base of the natural logarithm ($\approx$2.71828)

a, b, c = parameters to be estimated.

The height-diameter model is used to predict tree height from field measurement of tree diameter at breast height outside bark. It has been incorporated into all the tables formulated for individual tree volume estimation. A detailed description of the development of the height-diameter equations is presented in Report #2, Ecologically based individual tree height-diameter models for major Alberta tree species. Estimated height-diameter coefficients according to natural regions are listed in Appendix 2.

2.4 The Stump Diameter and Breast Height Diameter Model

A quadratic model expressing stump diameter outside bark as a function of breast height diameter outside bark is fitted on the provincial stem analysis data:

$$[6] \qquad DOB_{stp} = a + bD + cD^{2}$$

where:

DOB_{stp} = stump diameter outside bark (cm) at 0.30 m stump height

D = diameter at breast height outside bark (cm)

a, b, c = parameters to be estimated.

The regression function between stump diameter and breast height diameter is commonly used for conversion between these two diameters. It can be used to predict stump diameter from field measurement of breast height diameter, or to predict breast height diameter from field measurement of stump diameter. For the latter, equation [6] can be rearranged as follows:

$$[7] \qquad D = \frac{-b+\sqrt{b^2-4c(a-DOB_{stp})}}{2c}$$

As will be shown later, the relationship between stump diameter and breast height diameter allows prediction of individual tree volumes from stump diameter or breast height diameter. The estimated coefficients for the stump diameter and breast height diameter model, according to natural regions, are listed in Appendix 2.

3.0 METHODS OF COMPUTATION

Tables presented in subsequent reports in this series were constructed using the statistical relationships described above. Examples of the computations for variables such as individual tree gross total volume, gross merchantable volume, merchantable length, trees/m^3 merchantable volume, and others are discussed in the following sections. Figure 2 provides a simple graphic illustration for all computations described hereafter.

3.1 Merchantable Length

Merchantable length (*ML*) which extends from stump height to the height of a specified top diameter inside bark (*d*), is calculated using the taper equation. The calculations involve the following steps:

1. Specify the top diameter inside bark, *d*. In Alberta, *d* values of 5, 7, 10, 11, 13 and 15 cm are most commonly used.

2. Rearrange equation [1] into:

$$[8] \qquad h/H = \left[1-\left(\frac{d}{k}\right)^{1/c}(1-\sqrt{p})\right]^2$$

where k and c are defined in equations [9] and [10], respectively:

$$[9] \qquad k = a_0 D^{a_1} a_2^{D}$$

$$[10] \qquad c = b_1(h/H)^2 + b_2\ln(h/H+0.001) + b_3\sqrt{h/H} + b_4 e^{h/H} + b_5(D/H)$$

3. Use a mathematical iteration routine to calculate merchantable height (*MH*).

In equation [8], d is set to the specified top diameter (e.g., d = 7.0 cm) and c in [10] is calculated from a guessed initial value of h/H, termed $(h/H)_0$. A good initial value for $(h/H)_0$ for all Alberta tree species is 0.9. Once the c is calculated using the initial value, the first estimation of h/H, termed $(h/H)_1$ is obtained from equation [8]. Having calculated $(h/H)_1$, the next h/H value, termed $(h/H)_2$, used to

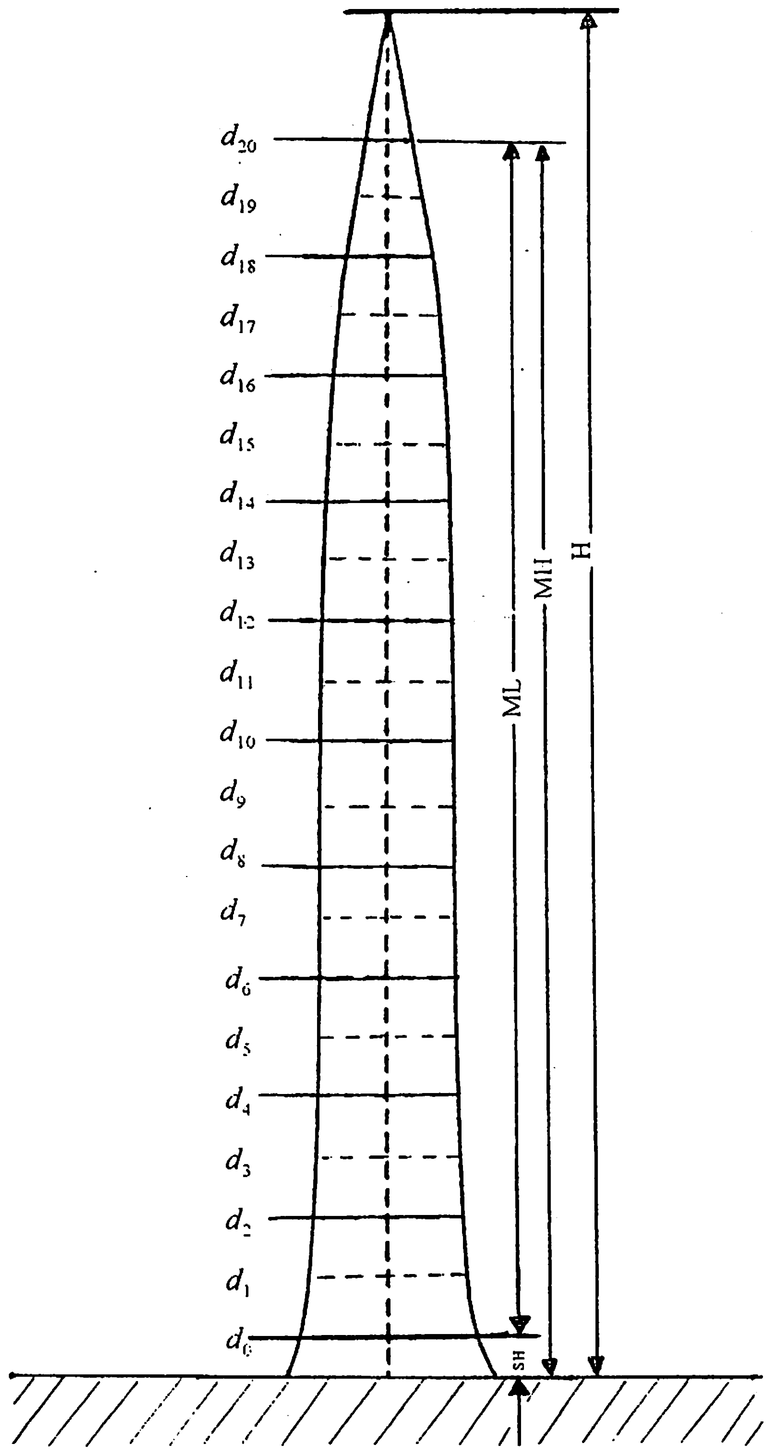

Figure 2. A graphic illustration for tree volume calculations. Where d_i indicates diameter inside bark along the stem, *ML* is merchantable length, *MH* is merchantable height, *SH* is stump height, and *H* is total tree height.

estimate c in equation [10] can be estimated using equation [11]:

$$[11] \qquad (h/H)_2 = \frac{(h/H)_1 + (h/H)_0}{2}.$$

This process is repeated until a desired precision is obtained, for example:

$$[12] \qquad |(h/H)_i - (h/H)_{i-1}| < 0.00000001$$

The above calculations can be readily programmed using the Statistical Analysis System (SAS). For instance, the following SAS statements will perform the iteration:

```
*This program calculates the merchantable length of the tree;
*Generate a set of trees with dbh from 8 to 80 by 2 cm, total tree height (H) from 4 to 40 by 2 m;

        data v1;
                do dbh = 8 to 80 by 2;
                   do H = 4 to 40 by 2; output;
                   end;
                end;
        run;

*The iteration procedure, using estimated coefficients for softwood from natural regions 2, 15, and 16 (see Appendix 2, Table A2);
*A 7.0 cm top diameter inside bark is assumed;

        data v2;
               set v1;
               a0 = 0.858012;  a1 = 0.994667;  a2 = 0.998503;  b1 = 0.957817;
               b2 = -0.228150;  b3 = 1.696453;  b4 = -0.788021;  b5 = 0.142355;

*Define z=MH/H, set the initial value for z;

               z0 = 0.9;

*The following iteration process is repeated until a desired precision is obtained;

               do until(abs(z0-z1) < 0.00000001);
                    c = b1*(z0)**2+b2*log(z0+0.001)+b3*sqrt(z0)+b4*exp(z0)+b5*(dbh/H);
                    z1 = (1-(( 7/(a0*dbh**a1*a2**dbh))**(1/c))*(1-sqrt(0.225)))**2;
                    z0 = (z0+z1)/2;
               end;

*Keep the coefficients and the final z0 or z1;

               keep a0-a2 b1-b5 dbh H z0 z1;
        run;
```

Once the specified precision is obtained, merchantable height (*MH*) of the tree is calculated according to: $MH = z0 \times H$, where H is the total tree height. Merchantable length (ML) of the tree is MH minus stump height (SH), that is, $ML = MH - SH$. A stump height value of 0.30 metres is most commonly used in Alberta. The tree dimensions of MH, SH, H and ML are easily discernable from Figure 2. See Appendix 3 for the calculations.

3.2 Gross Merchantable Volume

Once merchantable length (ML) has been calculated, gross merchantable volume can be calculated through a four-step procedure (see Figure 2):

1. Divide merchantable length into 10 sections of equal length.

2. Compute the height above the ground from the middle and the top of each section. For example, for the section next to the stump, its midpoint (corresponding to d_1) occurs at a height termed h_1, which is calculated as follows:

[13] $$h_1 = 1 \times ML/20 + SH$$

where ML and SH are merchantable length and stump height, respectively. The height above the ground at the top of the same section (corresponding to d_2) is termed h_2 and is calculated as follows:

[14] $$h_2 = 2 \times ML/20 + SH$$

It is quite clear that heights above the ground for other sections can be calculated in a similar manner, using the following generalized equation:

[15] $$h_i = i \times ML/20 + SH$$

Since there are 10 sections, $i = 1, 2, \ldots, 20$. Height above the ground at which the specified top diameter inside bark ($d = d_{20}$) occurs is equal to the merchantable height, that is, $h_{20} = 20 \times ML/20 + SH = MH$. In total, 20 calculations of heights above the ground are obtained from step (2).

3. Diameters inside bark at the middle and the top of each section are calculated using the taper equation,

expressed as follows:

$$[16] \qquad d_i = a_0 D^{a_1} a_2^{D} \left(\frac{1-\sqrt{h_i/H}}{1-\sqrt{p}}\right)^{b_1(h_i/H)^2 + b_2 \ln(h_i/H+0.001) + b_3\sqrt{h_i/H} + b_4 e^{h_i/H} + b_5(D/H)}.$$

There are 20 (i = 1, 2, ... , 20) diameters inside bark, d_1, d_2, d_3, ..., d_{20}, to be predicted at this stage. Diameter inside bark at the top of the stump, termed d_0, is also predicted from the taper equation with h_i equal to the stump height.

4. Merchantable volume is calculated using Newton's formula (Husch et al. 1982). There are 21 diameters, d_0, d_1, d_2, ..., d_{20}, located at intervals of $ML/20$ m, from top of the stump to the point where the minimum top diameter d ($d = d_{20}$) is specified (see Figure 2). Three diameters are required to compute the volume for each section of $2 \times ML/20$ (= $ML/10$) length. Thus, using Newton's formula the gross total merchantable volume of the tree is the summation of the volumes from 10 sections:

$$\begin{aligned}
[17] \qquad V_m = {} & \frac{ML/10}{6}(0.00007854)(d_0^2+4d_1^2+d_2^2) \\
& + \frac{ML/10}{6}(0.00007854)(d_2^2+4d_3^2+d_4^2) \\
& + \frac{ML/10}{6}(0.00007854)(d_4^2+4d_5^2+d_6^2) \\
& + \frac{ML/10}{6}(0.00007854)(d_6^2+4d_7^2+d_8^2) \\
& + \frac{ML/10}{6}(0.00007854)(d_8^2+4d_9^2+d_{10}^2) \\
& + \frac{ML/10}{6}(0.00007854)(d_{10}^2+4d_{11}^2+d_{12}^2) \\
& + \frac{ML/10}{6}(0.00007854)(d_{12}^2+4d_{13}^2+d_{14}^2) \\
& + \frac{ML/10}{6}(0.00007854)(d_{14}^2+4d_{15}^2+d_{16}^2) \\
& + \frac{ML/10}{6}(0.00007854)(d_{16}^2+4d_{17}^2+d_{18}^2) \\
& + \frac{ML/10}{6}(0.00007854)(d_{18}^2+4d_{19}^2+d_{20}^2)
\end{aligned}$$

where V_m is the merchantable volume (m^3) of the tree to the specified top diameter inside bark, ML is merchantable length (m), d_0, d_1, d_2, ..., d_{20} are diameters inside bark (cm) along the stem.

3.3 Gross Total Volume

Gross total volume of the tree is calculated by

$$[18] \qquad V = V_m+V_t+V_s$$

where:

V = gross total volume of the tree (m^3)

V_m = gross merchantable volume (m^3) of the tree to a specified top diameter

V_t = tip volume (m^3)

V_s = stump volume (m^3).

The following steps are necessary in computing gross total volume of the tree:

1. Specify the top diameter inside bark, then calculate the gross merchantable volume of the tree (V_m) using previously described procedures. In consultation with Land and Forest Services personnel, a 2.0 cm top diameter inside bark is specified for all Alberta tree species at this step.

2. Calculation of the tip volume. The tip of the tree is assumed to be a cone, so tip volume is calculated using the equation for a cone:

$$[19] \qquad V_t = \pi(d/200)^2(H-MH)/3$$

where:

V_t = tip volume (m^3)

$H - MH$ = tip length (m)

H = total tree height (m)

MH = merchantable height (m) to the specified top diameter (d = 2.0 cm)

d = top diameter inside bark (d = 2.0 cm).

3. Calculation of the stump volume. The stump of the tree is assumed to be a cylinder, so stump volume is calculated using the equation for a cylinder:

$$[20] \quad V_s = \pi (d_0/200)^2 SH$$

where:

V_s = stump volume (m^3)

d_0 = stump diameter inside bark (cm) predicted from the taper equation

SH = stump height (m).

The summation of the calculated merchantable volume, tip volume and stump volume gives the gross total volume (m^3) of the tree. See Appendix 3 for the calculations.

3.4 Trees/m³ Merchantable Volume

Number of trees per cubic metre of merchantable volume is calculated using the following equation:

$$[20] \quad Trees/m^3 = 1/V_m$$

where V_m is the merchantable volume (m^3) of the tree to the specified top diameter inside bark.

Once the coefficients have been estimated, computations for merchantable length, merchantable volume, and trees/m³ merchantable volume depend on the choices of top diameters. Tables displaying values for these variables were formulated using top diameters of 5, 7, 10, 11, 13 and 15 cm. Computations for total tree volume require "sectioning" the tree into three main portions (merchantable, tip and stump), and calculating the volume for each portion using the pertinent formulas. It should be remembered that in computing volume for the merchantable portion of a tree, the top diameter inside bark of 2.0 cm is consistently used for all Alberta tree species.

3.5 Stump Diameter Outside Bark

The regression function between stump diameter and breast height diameter, equation [6], can be used to predict stump diameter outside bark from field measurement of breast height diameter outside bark. Stump diameter outside bark can also be predicted using the taper equation and the diameter outside/inside bark relationship. For example, using the taper equation, diameter inside bark at stump height can be predicted, and this predicted value can be used to predict stump diameter outside bark with the diameter outside/inside bark relationship.

The two stump diameters obtained from the two different approaches will differ. For this study, equation [6] was used to predict stump diameter outside bark from breast height diameter outside bark. Reasons for not using the second approach included the following: (1) the first approach is simple and straightforward with high precision (see fit statistics in Appendix 2), (2) the taper equation is less reliable for predicting diameters at stump or top of the tree. A limitation to the use of equation [6] is that it can predict stump diameter outside bark only at stump height of 0.3 m. For any other stump height defined by the user, taper equation and diameter outside/inside bark equation must be used.

Computations for other variables such as diameter outside and inside bark, and total tree height only require fitted base models. Appendix 3 provides an example of a SAS program illustrating all computations discussed above.

4.0 FORMULATION OF THE TABLES

4.1 The Format

The tables were designed to be easily understood and applied, while the information contained in each table was maximized. A typical example of the gross total volume table is shown in Table 1. Compared to conventional volume tables, natural region based individual tree volume tables developed in this study represent expanded versions of volume tables that incorporate the use of the height-diameter relationship and permit a choice of having either the breast height diameter or the stump diameter as the main input variable.

Each type of table is distinguished by its title (e.g., from Table 1, one can easily tell that the table is for the gross total volume). Printed on the top left-hand side of each table are the applicable species and natural regions. The first two columns of the table are diameter at breast height outside bark (DBHOB) and diameter at stump outside bark (STUMP DOB). A stump height of 0.3 m is consistently used for all Alberta tree species.

All tables were initially created using values corresponding to the midpoints of 2.0 cm diameter classes and 2.0 m height classes. The DBHOB column is arranged by intervals of 2.0 cm diameter classes. To facilitate reading of the table, a fixed range is used instead of a single class value. For example, for the 2.0 cm DBHOB class, a range of 1.1 cm to 3.0 cm is used, and for the 20.0 cm DBHOB class, a range of 19.1 cm to 21.0 cm is used. In both cases, 2.0 cm and 20.0 cm are considered the midpoints of the classes.

The two boundary values for STUMP DOB corresponding to each DBHOB class are predicted with the stump diameter and breast height diameter relationship, using the two boundary values of the DBHOB class as the inputs. The main body of the table follows the first 2 columns. Each column is arranged by intervals of 2.0 m height classes up to 40.0 m. The far right-hand column of the table list the heights predicted from the midpoints of DBHOB classes.

Table 1. An example of a natural region based volume table

ALBERTA LAND AND FOREST SERVICES —— INDIVIDUAL TREE VOLUME TABLE (1994)

Gross total volume (m^3) from 0.00 m stump height to 0.0 cm top dib

SPECIES: SOFTWOOD
NATURAL REGIONS: 2, 15, 16

DBHOB (cm)	STUMP DOB (cm)	TOTAL TREE HEIGHT (m) 3.1-5.0	5.1-7.0	7.1-9.0	9.1-11.0	11.1-13.0	13.1-15.0	15.1-17.0	17.1-19.0	19.1-21.0	21.1-23.0	23.1-25.0	25.1-27.0	27.1-29.0	29.1-31.0	31.1-33.0	33.1-35.0	35.1-37.0	37.1-39.0	39.1-41.0	Predicted HT
1.1- 3.0	2.2- 4.1	0.0004	0.0007	0.0009	0.0011	0.0013	0.0015	0.0017	0.0020	0.0022	0.0024	0.0026	0.0028	0.0031	0.0033	0.0035	0.0037	0.0039	0.0042	0.0044	2.2
3.1- 5.0	4.2- 6.2	0.0022	0.0033	0.0045	0.0057	0.0068	0.0080	0.0092	0.0104	0.0116	0.0127	0.0139	0.0151	0.0163	0.0175	0.0186	0.0198	0.0210	0.0222	0.0234	3.9
5.1- 7.0	6.3- 8.2	0.0047	0.0073	0.0100	0.0126	0.0153	0.0179	0.0206	0.0232	0.0259	0.0285	0.0312	0.0339	0.0365	0.0392	0.0419	0.0446	0.0472	0.0499	0.0526	5.9
7.1- 9.0	8.3- 10.3	0.0081	0.0127	0.0173	0.0220	0.0267	0.0313	0.0360	0.0407	0.0454	0.0501	0.0548	0.0595	0.0642	0.0690	0.0737	0.0784	0.0831	0.0878	0.0926	8.0
9.1-11.0	10.4- 12.4	0.0124	0.0194	0.0265	0.0337	0.0409	0.0482	0.0554	0.0627	0.0700	0.0773	0.0845	0.0918	0.0992	0.1065	0.1138	0.1211	0.1284	0.1357	0.1431	10.2
11.1-13.0	12.6- 14.6	0.0175	0.0274	0.0375	0.0477	0.0580	0.0683	0.0786	0.0890	0.0994	0.1098	0.1202	0.1306	0.1410	0.1515	0.1619	0.1724	0.1828	0.1933	0.2038	12.3
13.1-15.0	14.7- 16.7	0.0234	0.0366	0.0501	0.0638	0.0776	0.0915	0.1055	0.1194	0.1334	0.1475	0.1615	0.1756	0.1897	0.2038	0.2179	0.2320	0.2461	0.2603	0.2744	14.3
15.1-17.0	16.8- 18.9	0.0301	0.0471	0.0644	0.0821	0.0999	0.1178	0.1358	0.1539	0.1720	0.1902	0.2084	0.2266	0.2448	0.2631	0.2814	0.2997	0.3180	0.3363	0.3546	16.1
17.1-19.0	19.0- 21.1	0.0377	0.0587	0.0803	0.1023	0.1246	0.1470	0.1696	0.1922	0.2150	0.2377	0.2606	0.2834	0.3063	0.3293	0.3522	0.3752	0.3982	0.4212	0.4442	17.8
19.1-21.0	21.2- 23.3	0.0462	0.0716	0.0978	0.1246	0.1517	0.1791	0.2067	0.2344	0.2621	0.2900	0.3179	0.3459	0.3740	0.4020	0.4301	0.4583	0.4864	0.5146	0.5428	19.4
21.1-23.0	23.4- 25.6	0.0556	0.0857	0.1169	0.1488	0.1812	0.2140	0.2469	0.2801	0.3134	0.3468	0.3803	0.4139	0.4475	0.4812	0.5149	0.5487	0.5825	0.6163	0.6502	20.8
23.1-25.0	25.7- 27.8	0.0660	0.1011	0.1376	0.1750	0.2130	0.2515	0.2903	0.3294	0.3687	0.4080	0.4476	0.4872	0.5269	0.5666	0.6064	0.6463	0.6862	0.7262	0.7662	22.1
25.1-27.0	28.0- 30.1	0.0775	0.1178	0.1598	0.2030	0.2471	0.2917	0.3368	0.3821	0.4278	0.4736	0.5195	0.5656	0.6118	0.6581	0.7044	0.7508	0.7973	0.8439	0.8904	23.2
27.1-29.0	30.2- 32.4	0.0900	0.1358	0.1836	0.2330	0.2834	0.3345	0.3862	0.4383	0.4906	0.5433	0.5961	0.6491	0.7022	0.7554	0.8087	0.8621	0.9156	0.9692	1.0228	24.2
29.1-31.0	32.5- 34.7	0.1038	0.1553	0.2091	0.2648	0.3219	0.3799	0.4385	0.4976	0.5572	0.6170	0.6771	0.7374	0.7978	0.8584	0.9191	0.9800	1.0409	1.1019	1.1629	25.1
31.1-33.0	34.9- 37.1	0.1188	0.1761	0.2362	0.2986	0.3626	0.4277	0.4937	0.5602	0.6273	0.6947	0.7625	0.8305	0.8986	0.9670	1.0355	1.1042	1.1729	1.2418	1.3107	26.0
33.1-35.0	37.2- 39.5	0.1351	0.1984	0.2649	0.3342	0.4054	0.4780	0.5516	0.6260	0.7009	0.7763	0.8521	0.9281	1.0045	1.0810	1.1577	1.2346	1.3116	1.3887	1.4660	26.7
35.1-37.0	39.6- 41.8	0.1529	0.2223	0.2954	0.3718	0.4505	0.5308	0.6124	0.6948	0.7780	0.8617	0.9458	1.0303	1.1152	1.2002	1.2855	1.3710	1.4567	1.5425	1.6284	27.3
37.1-39.0	42.0- 44.2	0.1722	0.2477	0.3276	0.4113	0.4977	0.5860	0.6758	0.7667	0.8584	0.9507	1.0436	1.1369	1.2306	1.3246	1.4189	1.5133	1.6080	1.7029	1.7979	27.9
39.1-41.0	44.4- 46.7	0.1932	0.2749	0.3616	0.4527	0.5470	0.6437	0.7420	0.8415	0.9421	1.0434	1.1453	1.2478	1.3507	1.4540	1.5575	1.6614	1.7655	1.8697	1.9742	28.4
41.1-43.0	46.8- 49.1	0.2158	0.3038	0.3974	0.4961	0.5986	0.7037	0.8108	0.9193	1.0290	1.1396	1.2509	1.3629	1.4753	1.5882	1.7014	1.8150	1.9288	2.0429	2.1571	28.8
43.1-45.0	49.2- 51.6	0.2404	0.3346	0.4351	0.5416	0.6523	0.7661	0.8822	1.0000	1.1191	1.2393	1.3603	1.4820	1.6044	1.7272	1.8504	1.9740	2.0979	2.2221	2.3466	29.2
45.1-47.0	51.7- 54.1	0.2669	0.3673	0.4748	0.5890	0.7082	0.8309	0.9563	1.0835	1.2123	1.3424	1.4734	1.6052	1.7377	1.8708	2.0044	2.1383	2.2727	2.4074	2.5423	29.6
47.1-49.0	54.2- 56.6	0.2956	0.4020	0.5164	0.6386	0.7664	0.8982	1.0329	1.1699	1.3087	1.4488	1.5901	1.7323	1.8753	2.0189	2.1631	2.3078	2.4529	2.5984	2.7442	29.9
49.1-51.0	56.7- 59.1	0.3265	0.4389	0.5602	0.6903	0.8267	0.9678	1.1121	1.2591	1.4080	1.5586	1.7104	1.8632	2.0170	2.1715	2.3266	2.4823	2.6385	2.7951	2.9521	30.1
51.1-53.0	59.2- 61.7	0.3598	0.4779	0.6061	0.7441	0.8894	1.0398	1.1940	1.3511	1.5104	1.6715	1.8341	1.9979	2.1627	2.3284	2.4948	2.6618	2.8293	2.9973	3.1658	30.4
53.1-55.0	61.8- 64.2	0.3957	0.5193	0.6542	0.8002	0.9543	1.1142	1.2784	1.4458	1.6158	1.7877	1.9614	2.1363	2.3125	2.4895	2.6674	2.8460	3.0252	3.2049	3.3851	30.6
55.1-57.0	64.4- 66.8	0.4344	0.5631	0.7045	0.8585	1.0216	1.1911	1.3653	1.5433	1.7240	1.9071	2.0920	2.2784	2.4660	2.6548	2.8444	3.0349	3.2260	3.4178	3.6101	30.8
57.1-59.0	67.0- 69.4	0.4760	0.6096	0.7573	0.9192	1.0912	1.2703	1.4549	1.6435	1.8352	2.0295	2.2259	2.4240	2.6234	2.8241	3.0258	3.2284	3.4317	3.6358	3.8404	30.9
59.1-61.0	69.6- 72.1	0.5207	0.6586	0.8125	0.9822	1.1631	1.3521	1.5470	1.7464	1.9493	2.1551	2.3631	2.5731	2.7846	2.9974	3.2114	3.4263	3.6422	3.8587	4.0760	31.1
61.1-63.0	72.2- 74.7	0.5688	0.7106	0.8703	1.0477	1.2376	1.4363	1.6417	1.8521	2.0663	2.2837	2.5036	2.7256	2.9493	3.1746	3.4011	3.6287	3.8572	4.0866	4.3167	31.2
63.1-65.0	74.9- 77.4	0.6204	0.7654	0.9307	1.1156	1.3144	1.5231	1.7390	1.9604	2.1861	2.4153	2.6473	2.8815	3.1177	3.3556	3.5948	3.8353	4.0768	4.3192	4.5624	31.3
65.1-67.0	77.5- 80.1	0.6758	0.8234	0.9938	1.1862	1.3938	1.6124	1.8389	2.0715	2.3088	2.5499	2.7941	3.0408	3.2897	3.5403	3.7925	4.0461	4.3007	4.5565	4.8131	31.4
67.1-69.0	80.2- 82.8	0.7352	0.8847	1.0598	1.2593	1.4758	1.7042	1.9414	2.1852	2.4343	2.6875	2.9440	3.2034	3.4651	3.7287	3.9941	4.2609	4.5290	4.7983	5.0685	31.5
69.1-71.0	82.9- 85.5	0.7989	0.9494	1.1287	1.3352	1.5604	1.7987	2.0466	2.3017	2.5626	2.8280	3.0971	3.3692	3.6439	3.9208	4.1995	4.4798	4.7616	5.0445	5.3285	31.5
71.1-73.0	85.7- 88.3	0.8671	1.0176	1.2007	1.4139	1.6476	1.8957	2.1543	2.4209	2.6937	2.9714	3.2532	3.5383	3.8261	4.1164	4.4086	4.7027	4.9982	5.2951	5.5931	31.6
73.1-75.0	88.4- 91.1	0.9403	1.0896	1.2758	1.4954	1.7376	1.9955	2.2648	2.5428	2.8276	3.1178	3.4123	3.7105	4.0117	4.3155	4.6215	4.9293	5.2389	5.5499	5.8622	31.7
75.1-77.0	91.2- 93.9	1.0185	1.1656	1.3542	1.5798	1.8303	2.0979	2.3780	2.6675	2.9643	3.2670	3.5745	3.8858	4.2005	4.5180	4.8379	5.1598	5.4835	5.8089	6.1356	31.7
77.1-79.0	94.0- 96.7	1.1023	1.2457	1.4360	1.6673	1.9258	2.2030	2.4938	2.7949	3.1038	3.4191	3.7396	4.0643	4.3926	4.7239	5.0578	5.3940	5.7321	6.0719	6.4133	31.8
79.1-81.0	96.8- 99.5	1.1919	1.3302	1.5213	1.7579	2.0242	2.3110	2.6124	2.9250	3.2461	3.5741	3.9076	4.2458	4.5878	4.9331	5.2812	5.6318	5.9844	6.3389	6.6951	31.8

UNDERLINED VALUES IN THE MIDDLE PORTION OF THE TABLE REPRESENT VOLUMES FOR AVERAGE HEIGHT-DIAMETER TREES.

Underlined values in the middle portion of each table represent the average height-diameter relationship. Values in two corners of the tables indicate unlikely tree sizes. Tables or matrices of user-defined ranges can be readily created using the actual functions (see Appendix 2 for coefficients). Assistance with re-creating similar tables is available upon request.

4.2 Applications of the Tables

The format of each table represents an integrated use of the taper equation and the relationships between tree height and tree diameter, diameter outside bark and inside bark, stump diameter and breast height diameter. In addition to its use as a standard volume table, the sample volume table (Table 1) can also be used as a local volume table, a height-diameter prediction table, and a stump diameter and breast height diameter prediction table. Here are some of the most common uses of such a table:

1. Prediction of total tree height. From field measurement of tree diameter at breast height outside bark (DBHOB), total tree height (HT) can be predicted from the far right hand side column of the table. For example, if a tree has a DBHOB of 29.4 cm, predicted HT of the tree from Table 1 is 25.1 m.

2. Prediction of stump diameter outside bark (STUMP DOB). From field measurement of DBHOB, STUMP DOB can be predicted. For example, if a tree has a DBHOB of 29.4 cm, then the predicted STUMP DOB of the tree from Table 1 falls between 32.5 cm and 34.7 cm.

3. Prediction of tree diameter at breast height outside bark (DBHOB). From field measurement of STUMP DOB, DBHOB can be predicted. For example, if a tree has a STUMP DOB of 44.5 cm, predicted DBHOB of the tree from Table 1 falls between 39.1 cm and 41.0 cm.

4. Prediction of gross total volume from observed DBHOB and HT. Gross total volume (m^3) to 0.0 cm top diameter inside bark (d) and 0.00 m stump height can be read directly from Table 1, based on the field measurements of total tree height and diameter at breast height outside bark. For example, if a 29.4 cm (DBHOB) tree has a HT of 21.8 m, the gross total volume of the tree from Table 1 is 0.6170 m^3.

5. Prediction of gross total volume from observed DBHOB and predicted HT. If the field measurement of tree height is not available, the far right-hand column of the table can be used to predict HT from field measurement of DBHOB. Based on the DBHOB and the predicted HT, gross total volume can be read from the table. For example, if a tree has a DBHOB of 29.4 cm, then the predicted HT of the tree will be 25.1 m. The gross total cubic metre volume of the tree from Table 1 is 0.7374 m^3, which is underlined for easy reading. In the absence of field measurement for tree heights, only the underlined values in the middle portion of the table, which represent the result of the average height-diameter relationship, are needed for estimating the gross total volumes (m^3) of the tree.

6. Prediction of gross total volume from observed stump diameter and HT. The second column from the left, in the standard table for gross total volume, represents stump diameter outside bark. If the field measurements of stump diameter and total tree height are available, gross total volume can be read from Table 1. For example, if a tree has a STUMP DOB of 23.2 cm and a HT of 15.8 m, the gross total volume of the tree from Table 1 is 0.2067 m^3.

7. Prediction of gross total volume from observed stump diameter and predicted HT. If the field measurement of stump diameter is all that is available, the far right-hand column of the table can be used to predict HT from STUMP DOB. Based on the STUMP DOB and the predicted HT,

gross total volume (m^3) can be read from the table. For example, if a tree has a STUMP DOB of 23.2 cm, predicted HT of the tree from the far right-hand column of Table 1 is 19.4 m, and the gross total volume of the tree from Table 1 is 0.2621 m^3. Once again, in the absence of field measurements for tree height, only the underlined values in the middle portion of the table, which represent the average height-diameter relationship, are needed for estimating the gross total volume.

Tables for predicting merchantable volume, merchantable length, and trees/m^3 merchantable volume can be used in a similar manner.

8. Prediction of log volume. Tables for predicting total volume, merchantable volume, and merchantable length can be used jointly to determine log volume. The procedure is described in Reports 4 to 11 of this series.

5.0 STATISTICS IN INDIVIDUAL TREE VOLUME ESTIMATION

Although the fitting of the equations and the construction of the tables appear relatively clear and straightforward, ecologically based individual tree volume estimation involves a number of rather intricate statistical concepts and problems that readers may not be readily aware of. They include:

5.1 The Choice of the Taper Equation

Kozak's (1988) variable-exponent taper equation was fitted for all Alberta tree species. A large number of published taper equations, including those most recently presented by Flewelling (1993), Flewelling and Raynes (1993), Newnham (1992), Perez et al. (1990), and those previously studied by the Alberta Forest Service (Alberta Forest Service 1987), were compared based on the plots of residuals and a certain number of fit statistics such as the mean squared errors, the coefficients of determination, and the average biases. Kozak's (1988) variable-exponent taper equation was determined to be the preferred model because variables required to fit the model were readily available from the provincial stem analysis data base, and were consistent with the data collection process in Alberta. The model also showed very high accuracies in predicting diameters inside bark along the stem for all Alberta tree species, and was relatively easy to fit.

However, a few cautionary notes about the fit of Kozak's taper equation should be mentioned. These problems involve methods for calculating merchantable height, use of logarithmatic transformation (if any) and its associated error correction, and different ways for making volume predictions. Appendix 1 describes some of these problems and the corresponding solutions. The weaknesses of the Kozak's taper equation, such as the need for numerical iteration in calculating merchantable length and the lack of mathematical integration in estimating volume (see Kozak 1988; Kozak and Smith 1993), will likely cause some small discrepancies in predicting merchantable lengths and volumes, depending on the choices of top diameters and the assumed number of sections to be used in calculations. Nevertheless, these small discrepancies are less profound and can be ignored in practice. The problem of inherent dependence of the tree sectioning data, caused by taking several measurements from the same tree, is also ignored in fitting the taper equation.

5.2 Accuracy of the Volume Estimation

Using the actual tree volume calculated from Smalian's or Newton's formula and the predicted tree volume calculated from proposed volume and taper equations, many researchers have compared accuracies of volume estimations by different equations. Such approaches, however, might not be appropriate since the so called "actual" tree volume obtained with Smalian's or Newton's formula is not the real "actual" tree volume. In other words, in making such comparisons, one volume estimate is compared to a different volume estimate, rather than to the true volume of the tree.

The water displacement techniques discussed in detail by Martin (1984) and others provide the appropriate approaches for testing the accuracies of volume and taper equations. But such experiments require substantial resources including the use of some very specialized equipment (e.g., the xylometer), and are not very practical if a large number of samples are involved.

With the limited resources available, direct testing of the accuracy of the taper equation in predicting actual tree volume is not feasible at this time. However, using the procedures described by Kozak and Smith (1993), "actual" tree volumes calculated from the Smalian's formula were compared to those obtained from the taper equation for comparisons of the two estimates. Results indicated that, on average, volumes obtained from the Smalian's formula were approximately 2% to 5% more than those obtained from the taper equation. Since it is known that, for long sections, Smalian's formula usually overestimates tree volume (up to 12%, see Husch et al. 1982), volumes derived from the taper equation should generally be regarded as more accurate.

5.3 Comparison Among Natural Regions

Tables were not formulated for each natural region, but for groups of natural regions that have similar patterns of relationships. Statistical tests were conducted to verify that significant differences do exist from one natural region group to the next.

Comparison of the differences of height-diameter models among natural regions provides the guidance for all grouping of natural regions. The regression method of dummy variables (also called indicator variables or binary variables) is used to make such comparisons. Dummy variables are frequently applied to models that allow for behavioral differences in geographic regions (Neter et al. 1990; Judge et al. 1988). For example, for the simple linear model $y = a + bx$, a dummy variable version of the model for two natural regions can be written as $y = (a + a_1x_1) + (b + b_1x_1)x$; this equation is the full model, where the dummy variable x_1 is defined as $x_1 = 0$ if natural region = 1, and $x_1 = 1$ if natural region = 2. It is obvious that the dummy variable version of the model represents two models: 1) for natural region 1 where $x_1 = 0$, $y = a + bx$ (the reduced model); and 2) for natural region 2 where $x_1 = 1$, $y = (a + a_1) + (b + b_1)x$. Identity of the two regression models for two natural regions is tested by considering the following alternatives:

H_0: $a_1 = b_1 = 0$

H_a: not both $a_1 = 0$ and $b_1 = 0$

The appropriate test statistic, the extra sum of squares method (Neter et al. 1990), is given by

$$F^* = \frac{SSE(R) - SSE(F)}{df_R - df_F} \div \frac{SSE(F)}{df_F} \qquad [22]$$

where F^* follows the F distribution when H_0 is true. The degrees of freedom df_R and df_F are associated with the reduced and the full model error sums of squares ($SSE(R)$ and $SSE(F)$), respectively. The statistical decision rule is:

If $F^* \leq F(1-\alpha;\ df_R - df_F,\ df_F)$, conclude H_o

If $F^* > F(1-\alpha;\ df_R - df_F,\ df_F)$, conclude H_a

The principle of dummy variables for linear least squares estimation can be readily extended to nonlinear models presented for individual tree volume estimation. For example, consider the nonlinear height-diameter model [5], if the purpose is to test whether the difference of height-diameter relationships between natural regions (or natural region groups) 1 and 2 is significant, the dummy variable version of the full height-diameter model can be written as:

$$[23] \qquad H = 1.3+(a+a_1x_1)\,[1-e^{-(b+b_1x_1)D}]^{(c+c_1x_1)}$$

This six-parameter full model has the following error sum of squares:

$$[24] \qquad SSE(F) = SSE(a_1,b_1,c_1,a,b,c)$$

with $df_F = n - 6$ degrees of freedom associated with it. The dummy variable x_1 in the full model is defined as follows:

$$[25] \qquad x_1 = \begin{matrix} 1 & \textit{if natural region} = 2 \\ 0 & \textit{otherwise} \end{matrix}$$

The reduced model for natural region 1, for which $x_1 = 0$, is as follows:

$$[26] \qquad H = 1.3+a(1-e^{-bD})^c$$

The error sum of squares for this three-parameter, reduced model is:

$$[27] \qquad SSE(R) = SSE(a,b,c)$$

There are $df_R = n - 3$ degrees of freedom associated with this reduced model. Identity of the two height-diameter models for two natural regions is tested by considering the alternatives:

H_0: $a_1 = b_1 = c_1 = 0$

H_a: at least one of the equalities in H_0 is not true

The test statistic in this case becomes:

$$[28] \qquad F^* = \frac{SSE(a,b,c)-SSE(a_1,b_1,c_1,a,b,c)}{(n-3)-(n-6)} \div \frac{SSE(a_1,b_1,c_1,a,b,c)}{n-6}$$

To compute the test statistic here, both the full model and the reduced model are fitted to provide error sums of squares. Specifying the level of significance at 0.05 ($\alpha = 0.05$), if the calculated $F^* \leq F$

(0.95; 3, n - 6), then H_0 is true and the reduced model is appropriate for combined natural region groups; if $F^* > F$ (0.95; 3, n - 6), then H_a is true and separate models are required for separate natural regions.

The test just described can be conducted for each possible pair of natural region groups, if the differences of the height-diameter relationships are to be examined among three or more natural region groups. A more detailed description of the procedure is presented in Report #2 of this series, Ecologically based individual tree height-diameter models for major Alberta tree species. Grouping of the differences in geographic regions for the taper equation and other relationships can also be conducted in a similar manner. Results of all classifications were almost identical to those obtained for the height-diameter model. To facilitate the practical use of the tables, natural region groups classified for the height-diameter model were consistently used for all other relationships.

5.4 The Simultaneous Nature of Equations

Tables developed through this study are based on a system of compatible, interdependent and analytically related equations. Within such a system of equations, a variable appearing on the left-hand side of an equation can also appear on the right hand of another equation in the system. Simultaneous equation methodologies (see Gallant 1987; Judge et al. 1985, 1988; Kmenta 1986) may be considered theoretically more appropriate for estimating the structural parameters of the system, and examples of such approaches were adeptly illustrated by LeMay (1988, 1990). However, such estimation processes depend heavily on asymptotic approximations (especially if nonlinear models are involved), and in a number of related analyses, show very little substantial gain over the conventional ordinary least squares method applied to individual equations of the system (Huang 1992). Therefore, all equations in ecoregion-based individual tree volume estimation were fitted separately. Further studies on the dynamic nature of the taper equation and its relationship with other equations in an individual tree volume estimation system should prove very useful. The dependence problem of the tree sectioning data should also be addressed.

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APPENDICES

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Appendix 1.

Cautionary Note on Fitting Kozak's Variable-Exponent Taper Equation

ABSTRACT

Kozak's variable-exponent taper equation was fitted for major tree species in Alberta. Examination of the residual plots indicated that the taper equation in its nonlinear form with an additive error structure was more appropriate than the multiplicative error structure implied by the commonly used procedure of linearizing the equation and estimating coefficients using multiple linear least squares methods. The average bias caused by the logarithmic transformation was small but approximately two times as large as that when nonlinear regression was used to estimate parameters. Adjustment for the bias improved performance of the linearized model, but the nonlinear form with additive error terms eliminated inequality of error variance.

INTRODUCTION

In his instrumental work on taper equation, Kozak (1988) opened up a new trend in developing taper equations and as a result, greatly improved the accuracy and precision in estimating diameter inside bark at any point on the stem, merchantable volume to any top diameter and from any stump height, and individual log and whole tree volumes. Kozak's model is essentially an allometric function of the following form:

$$[A1] \qquad y = kx^{c}$$

where k is a constant and c is the exponent. The shapes of any solid of revolution obtained by rotating a curve of the general form described by [A1] around the x axis resemble the forms of the stems of trees, which are often assumed to be neiloids, cylinders, paraboloids and cones. Kozak directly and indirectly defined y, k, x and c in [A1] and came up with the following variable-exponent taper equation:

$$[A2] \qquad d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

where

$$[A3] \qquad X = (1-\sqrt{h/H})\,/\,(1-\sqrt{p})$$

and

d = diameter inside bark at h (cm)

h = height above the ground (m), $0 \leq h \leq H$

H = total tree height (m)

D = diameter at breast height outside bark (cm)

$Z = h / H$

p = location of the inflection point, assumed to be at 22.5% of total height above the ground

e = base of the natural logarithm (≈ 2.71828)

a_0, a_1, a_2, b_1, b_2, b_3, b_4, b_5 = parameters to be estimated.

The location of the inflection point, according to Perez et al. (1990), had little effect on the predictive properties of the model. A constant p value of 0.225 suggested by Kozak (1988) was used in this study.

In order to estimate the parameters, Kozak's variable-exponent taper model [A2] is usually linearized using a logarithmic transformation (Kozak 1988, 1991; Perez et al. 1990):

$$[A4] \qquad \ln(d) = \ln(a_0) + a_1 \ln(D) + \ln(a_2) D + b_1 \ln(X) Z^2 + b_2 \ln(X) \ln(Z+0.001)$$
$$+ b_3 \ln(X) \sqrt{Z} + b_4 \ln(X) e^{Z} + b_5 \ln(X) (D/H) .$$

The regression coefficients of this transformed equation can then be calculated using multiple linear least squares methods, and predicted diameters inside bark obtained by exponentiation of the fitted linear equation.

Fitting of the linearized Kozak's taper equation [A4] has become a standard procedure, both for application purposes (Kozak 1988, 1991; Perez et al. 1990) and for comparison with other types of taper equations (Flewelling and Raynes 1993; Newnham 1992). Previous analyses, however, did not deal with the implicit error structure of the model nor did they address the problem of bias and reduced accuracy caused by the use of a logarithmic transformation. The primary objectives of this appendix are to 1) identify the error structure of the model, 2) compare linear and nonlinear forms of Kozak's taper equation, and 3) determine the most appropriate model form to use for major Alberta tree species.

To facilitate the description of the analysis, Kozak's variable-exponent taper equation [A2] was written more compactly as follows:

$$[A5] \qquad d = kX^{c}$$

where X is defined in [A3], and

$$[A6] \qquad k = a_0 D^{a_1} a_2^{D}$$

$$[A7] \qquad c = b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)\ .$$

THE DATA

Since a consistent pattern of results was obtained for all major Alberta tree species, only a Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) data set, consisting of 77 stem analysis trees, was used to illustrate the essence of this analysis. According to Kozak (1988), this number of sample trees should be sufficient. Most of the Douglas-fir trees were felled and measured in the Montane natural region located in the foothills and major valleys of the Rocky Mountains in Alberta. Diameter at breast height outside bark (D) and total tree height (H) were recorded from each sample tree. Sections of the tree were cut at stump height (0.30 m), breast height (1.30 m), 2.80 m above the ground, and an equal length of

2.5 m thereafter to the top of the tree. Diameters inside and outside bark at the top of each section were measured. Stem analysis data obtained in this manner are not independent since several measures were taken from one tree. But this problem is less profound for nonlinear models with large sample sizes and is often ignored in practice.

RESULTS AND DISCUSSION

Ordinary least squares (OLS) estimation of Kozak's model [A4] was relatively simple and straightforward. It was accomplished using the PROC REG procedure on SAS/STAT software (SAS Institute Inc. 1985), by minimizing the error sum of squares, which follows:

$$[A8] \qquad \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

where:

y_i = observed $\ln(d_i)$

$\hat{y}_i$ = predicted $\ln(d_i)$

n = number of observations.

Table A1 lists estimated coefficients and other regression statistics for [A4]. A typical residual plot is displayed in Figure A1, which shows the residuals, calculated as the observed minus predicted values of $\ln(d_i)$, plotted against the predicted values of the dependent variable $\ln(d_i)$.

Kozak's taper model can also be fitted in its original nonlinear form shown in equation [A2] (or [A5]). This fitting was accomplished using the PROC NLIN procedure on SAS/STAT by minimizing the error sum of squares, which follows:

$$[A9] \qquad \sum_{i=1}^{n} (d_i - \hat{d}_i)^2$$

where: d_i = observed diameter inside bark

$\hat{d}_i$ = predicted diameter inside bark.

Table A1. Fit statistics for the taper model in different forms on Douglas-fir data

Equation	Parameter	Estimate	Std. Error	n	MSE	R^2	Ave. bias
[A4], [A10]	$\ln(a_0)$	-0.213876	0.128556	638	0.02067	0.9704	0.000000
	a_1	1.008833	0.061109				
	$\ln(a_2)$	-0.002333	0.002711				
	b_1	1.264797	0.484132				
	b_2	-0.259289	0.105896				
	b_3	1.506128	0.937017				
	b_4	-0.745722	0.523191				
	b_5	0.041639	0.016761				
[A2], [A11]	a_0	0.913153	0.069876	638	1.1199	0.9868	-0.009557
	a_1	0.964386	0.036868				
	a_2	0.998391	0.001275				
	b_1	1.386315	0.200658				
	b_2	-0.286495	0.047080				
	b_3	1.783899	0.472877				
	b_4	-0.916932	0.250627				
	b_5	0.058830	0.014518				
[A10] - on original data				638	1.1586	0.9863	0.016877

Note: n = number of observations, MSE, R^2 and Ave. bias are mean squared error, coefficient of determination and average bias calculated according to [A13], [A14] and [A15] respectively.

The Gauss-Newton iterative method as described in Gallant (1987) was applied, and estimates of the coefficients obtained from the linearized equation were provided as starting values in nonlinear least squares estimation to achieve fast convergence. Results of the nonlinear fit statistics are also listed in Table A1. A residual plot is displayed in Figure A2, which shows the residuals, calculated as the observed minus predicted values of d_i, plotted against the predicted values of the dependent variable d_i.

The two alternative approaches considered above for fitting of the Kozak's taper equation in its linearized or nonlinear form, although easily implemented, cannot both be correct. This is caused by the substantial difference in underlying error specification. Fitting of the linearized and nonlinear equations by the least squares principle assumes additive error terms, that is, equations [A4] and [A2] (written as

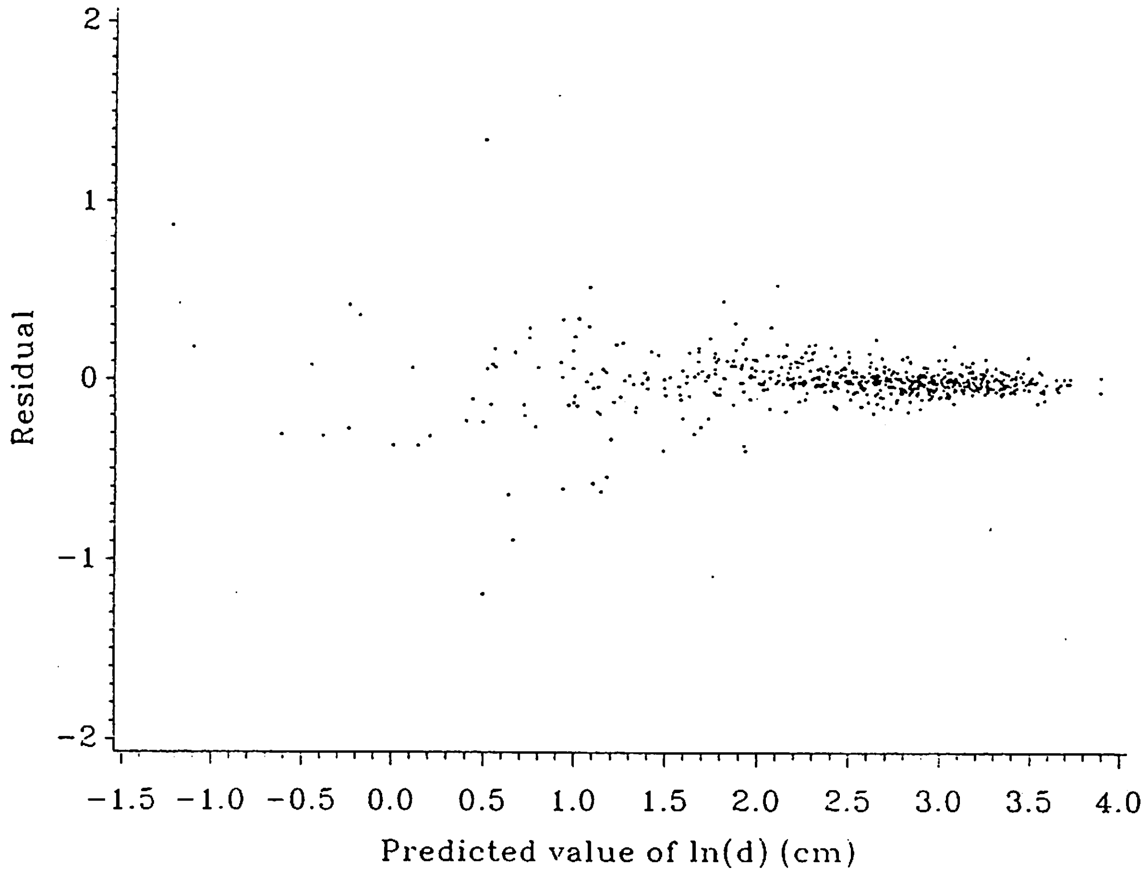

Figure A1. The plot of residuals [A4]. Residuals are calculated as the observed minus predicted values of $\ln(d_i)$.

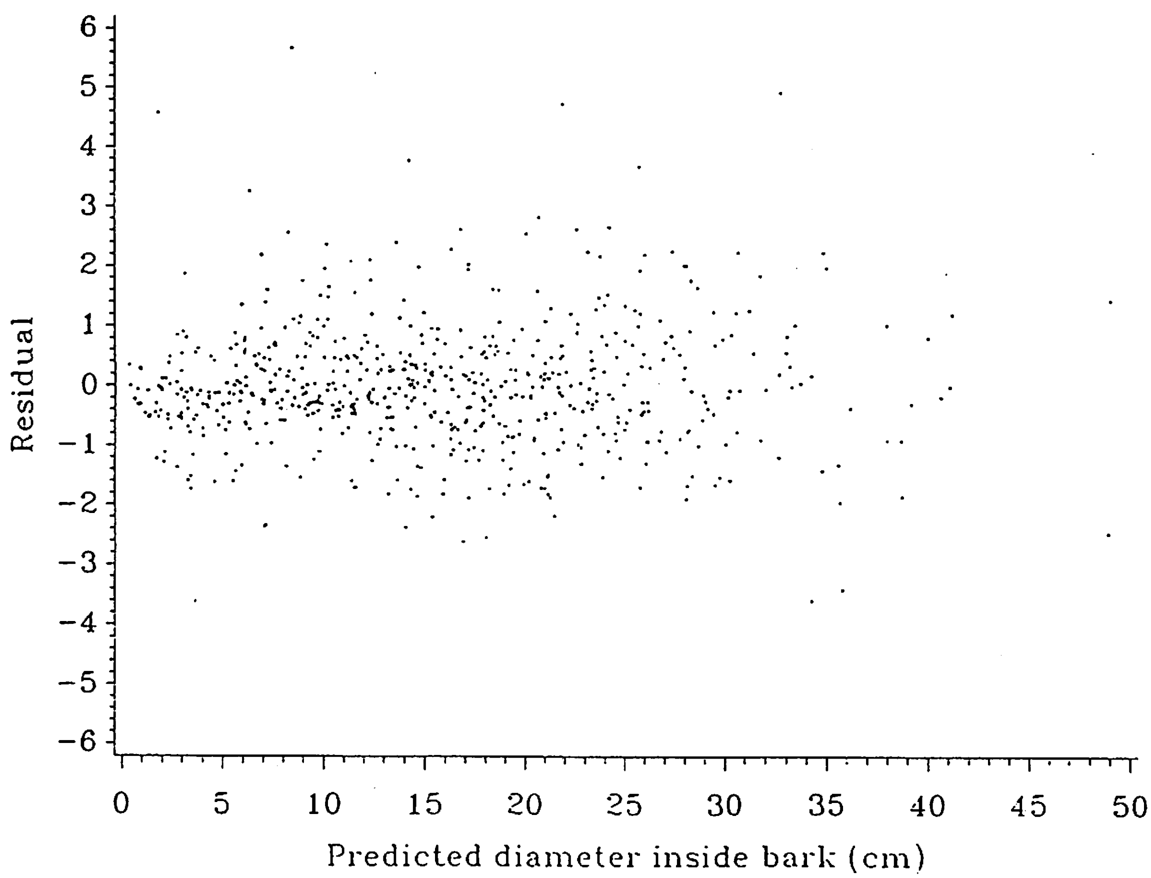

Figure A2. The plot of residuals from [A2]. Residuals are calculated as the observed minus predicted values of d_i.

[A5]) are estimated as follows, respectively:

$$[A10] \qquad \ln(d_i) = \ln(a_0)+a_1\ln(D)+\ln(a_2)D+b_1\ln(X)Z^2+b_2\ln(X)\ln(Z+0.01)$$
$$+b_3\ln(X)\sqrt{Z}+b_4\ln(X)e^{Z}+b_5\ln(X)(D/H)+\varepsilon_i$$

and

$$[A11] \qquad d_i = kX^{c}+\omega_i.$$

Notice however, that equations [A10] and [A11] are two very different statistical models, and their error terms (ε_i and ω_i) differ from one another. Equation [A10] is obtained by taking a logarithmic transformation of equation [A12]:

$$[A12] \qquad d_i = kX^{c}e^{\varepsilon_i}.$$

Equation [A11], on the other hand, cannot be linearized because the error term (ω_i) is additive rather than multiplicative as is the error term (ε_i) in equation [A12].

Thus, the proper identification of the error structure will determine whether [A11] or [A12], and consequently [A10], is adequate. If the model is [A11], then nonlinear least squares procedures are appropriate; if the model is [A12], multiple linear regression procedures can be used on equation [A10], provided that the necessary least squares assumptions regarding the residuals are met. Nonlinear least squares procedures can also be directly applied for [A12], if proper measures are taken to account for the multiplicative errors, such as the use of the weighted nonlinear least squares method (Gallant 1987; Judge et al. 1988).

Although more delicate procedures are available (see Judge et al. 1988), identification of the error structure is routinely conducted by simply examining the plot of residuals (or studentized residuals) that are calculated as the difference between the actual and predicted values of the dependent variable. Judging from the residual plots for this analysis, it is obvious that Figure A1, the residual plot from the linearized Kozak equation [A10], strongly indicates the problem of unequal variances of the error terms. On the other hand, Figure A2, the residual plot from Kozak's nonlinear equation [A11], shows a relatively constant error variance. It was therefore inferred that by linearization of the nonlinear equation [A2] without considering the error structure, an inappropriate regression problem of nonconstancy of error variance was created , which caused the estimated parameters to be inefficient (Judge et al. 1988, p. 341).

The satisfactory residual plot (Figure A2), which showed a constant error variance from Kozak's nonlinear model [A11], also indicates that the error structure of Kozak's taper equation is in fact, additive rather than multiplicative. Thus, the correct model is [A11], not the linearized form [A10] obtained from equation [A12].

Comparison of the Fitted Linear and Nonlinear Models

Mathematically, the correct equation [A11] cannot be linearized because the error term ω_i is additive rather than multiplicative. However, since the linearized Kozak's model [A10] was most commonly estimated in practice, it is interesting to see how well this linearized model, compared to the nonlinear model [A11], is in estimating the actual diameters inside bark d_i.

To do this, consider the fit statistics shown in Table A1, where the mean squared error (*MSE*), the coefficient of determination (R^2), and the average bias (B) were calculated for [A11] by

$$[A13] \qquad MSE = \frac{\sum_{i=1}^{n} (d_i - \hat{d}_i)^2}{n-m}$$

$$[A14] \qquad R^2 = 1 - \frac{\sum_{i=1}^{n} (d_i - \hat{d}_i)^2}{\sum_{i=1}^{n} (d_i - \bar{d}_i)^2}$$

$$[A15] \qquad B = \frac{\sum_{i=1}^{n} (d_i - \hat{d}_i)}{n}$$

where:

d_i = observed diameter inside bark

$\hat{d}_i$ = predicted diameter inside bark

$\bar{d}_i$ = observed average diameter inside bark

n = number of observations, and m = number of parameters (m = 8).

Fit statistics for equation [A10] in Table A1 were also calculated using [A13], [A14] and [A15], with the observed and predicted values of d_i replaced by the observed and predicted values of $\ln(d_i)$.

Since different dependent variables, d_i and $\ln(d_i)$, were used, it was not possible to make direct comparisons on the fit statistics for the purpose of assessing the performance of models [A11] and [A10]. Their goodness-of-fit on the original data, however, can be evaluated based on their precision (in terms of *MSE*) and accuracy (in terms of average bias) in estimating actual stem diameters inside bark. Fit statistics from equation [A11] are directly applicable to the original data since d_i is used as the dependent variable. Fit statistics from equation [A10] need to undergo exponentiation in order to make meaningful inferences about its precision and accuracy on the original data, since a transformed dependent variable, $\ln(d_i)$, is involved. A three-step procedure is commonly employed for equation [A10] so that it also applies for the original data:

1. Using equation [A10] and the estimated coefficients from Table A1, the predicted values of $\ln(d_i)$, $\widehat{\ln(d_i)}$, are obtained.

2. Take antilogs or exponentiation of the predicted values from the preceding step to give predicted $\hat{d}_i$ = $\exp[\widehat{\ln(d_i)}]$.

3. Calculate the mean squared error, the coefficient of determination (R^2), and the average bias according to [A13], [A14] and [A15], respectively, using the actual d_i from the data set and the predicted $\hat{d}_i$ from step 2.

Results of the calculated mean squared error, coefficient of determination, and average bias are also listed in Table A1 under the term "[A10] - on original data". Performance of model [A10] can now be directly compared to that of model [A11] based on these *MSE*, R^2, and *B* values calculated on the original data. It is clear that both *MSE* and R^2 values indicate the superiority of model [A11]. The absolute value of the average bias from model [A11], 0.009557, is only 0.06% of $\bar{d}_i$, the average observed diameter inside bark ($\bar{d}_i$ = 15.67735 cm). On the other hand, the absolute value of the average bias from model [A10], 0.016877, accounts for 0.11% of the average observed diameter inside bark, almost double the bias from model [A11], and therefore, indicates again that model [A11] provides a better fit on the original data.

Correction of Linearization Bias

The outcome of the above comparison is expected because of the use of a logarithmic transformation. When a transformation is applied to the dependent variable d_i, it is necessary to apply inverse transformation and express the predicted values of d_i in untransformed forms so that, for example, Newton's formula can be used to compute merchantable tree volume. Exponentiation of the fitted linear model, from a logarithmic scale back to the original scale without appropriate adjustment, can produce a severely biased model (Miller 1984) and affect the distribution of the predicted d_i values. These consequences are true even when the correct model is [A12] and the logarithmic transformation of the model, equation [A10], is appropriate.

The problem of the detransformation bias has been discussed by a number of researchers (Aitchison and Brown 1957; Bury 1975; Finney 1941; Miller 1984; SAS Institute Inc. 1988; Taylor 1986). Baskerville (1972) also presented a remedy to this problem in the estimation of plant biomass for balsam fir trees (*Abies balsamea* (L.) Mill.). A more detailed description of the problem was provided by Flewelling and Pienaar (1981). The fundamental reason for the exponentiation bias is that if the logarithmic transformation is appropriate, the dependent variable, $\ln(d_i)$, follows a normal distribution with mean μ and variance σ^2. However, it can be shown (Bury 1975 pp. 277-298; Meyer 1975 p. 285) that the distribution of d_i is log-normal with the mean

$$[A16] \quad E(d_i) = \exp(\mu+\sigma^2/2)$$

and the variance

$$[A17] \quad Var(d_i) = \exp(2\mu+\sigma^2)\,[\exp(\sigma^2)-1]\,.$$

The median of d_i is

$$[A18] \quad Median\ d_i = \exp(\mu)\,.$$

It is clear from [A18] that the predicted values for d_i obtained by exponentiation of $\widehat{\ln(d_i)}$, an unbiased estimator of μ from the linear least squares fit of [A10], are not unbiased estimates of the means of d_i, rather, they estimate the medians of d_i. Exponentiation of the fitted linear model thus characterizes the median rather than the mean value of d_i; This characterization causes a systematic underestimation of the mean response (Miller 1984). An average bias of +0.016877 (calculated according to [A15]) for model [A10], based on the detransformed data, confirms the notion of systematic underestimation.

Ignoring the dependence of the data, the biasing factor of $\exp(\sigma^2/2)$ from the logarithmic transformation is easily discernable from equations [A16] and [A18]. To adjust for this bias, approximately unbiased predicted values for d_i can be computed by

$$[A19] \quad \hat{d}_i = kX^c\exp(\hat{\sigma}^2/2)$$

where X is defined in [A3], k and c are defined in equations [A6] and [A7]. Coefficients in [A6] and [A7] are those obtained from the linear least squares fit of [A10]. The estimated error variance ($\hat{\sigma}^2 = MSE$) is also obtained from [A10].

Since the biasing factor $\exp(\sigma^2/2)$ is always greater than or equal to one [$\exp(\sigma^2/2) \geq 1$], the calculated values for $\hat{d}_i$ from [A19] are always greater than or equal to the $\hat{d}_i$ calculated without adjusting for the biasing factor. Thus, if $\hat{d}_i$ is not calculated according to [A19], a systematic underestimation of $\hat{d}_i$, and quite possibly the $\hat{d}_i$-based volumes, will occur. Kozak's (1988) results indicated that the biases for diameter inside bark and volume estimations showed a general trend of slight underestimation, and he suggested that this problem might be caused by the use of the Smalian's formula, which overestimates volume. Negative average biases between predicted and observed diameters inside bark (that is, $\hat{d}_i - d_i$) were also evident from the results obtained by Perez et al. (1990) on the independent testing data, indicating again that the diameters inside bark had been underestimated.

A possible addendum to Kozak's interpretation is that if the $\hat{d}_i$ values are calculated without adjustment for the biasing factor, diameters inside bark and, consequently, volumes will generally be underestimated. Perez et al.'s (1990) evidence of underestimations of diameters inside bark might also be caused by ignoring the biasing factor, rather than the true underestimation of the diameters by their model. Calculations of $\hat{d}_i$ according to [A19] may not eliminate all of the underestimations; however, it is the correct method given that the logarithmic transformation is assumed appropriate, and will at least make Kozak's taper equation shift towards the right direction. Exponentiation of the $\widehat{\ln(d_i)}$ from the least squares estimation of [A10] systematically underestimates the true diameters inside bark by the factor of $\exp(\sigma^2/2)$.

Kozak's Taper Equation for Major Alberta Tree Species

It should be emphasized that the above discussions are based on the premise that the logarithmic transformation of the Kozak's taper equation, shown in [A10] and commonly fitted by means of multiple linear regression packages, is correct. Under this assumption, predicted diameter inside bark should be computed according to [A19]. However, as judged from the residual plot in Figure A1, the linearized Kozak's taper equation [A10] is not appropriate in terms of the error specification for the Douglas-fir data set, nor as is shown later, for all other major tree species in Alberta.

The correct error structure, as judged from the residual plot in Figure A2, is described by [A11]. Hence, the nonlinear least squares estimation of [A11] is the appropriate procedure to use for Kozak's taper equation. Results of the nonlinear least squares fit statistics, along with those from linearized

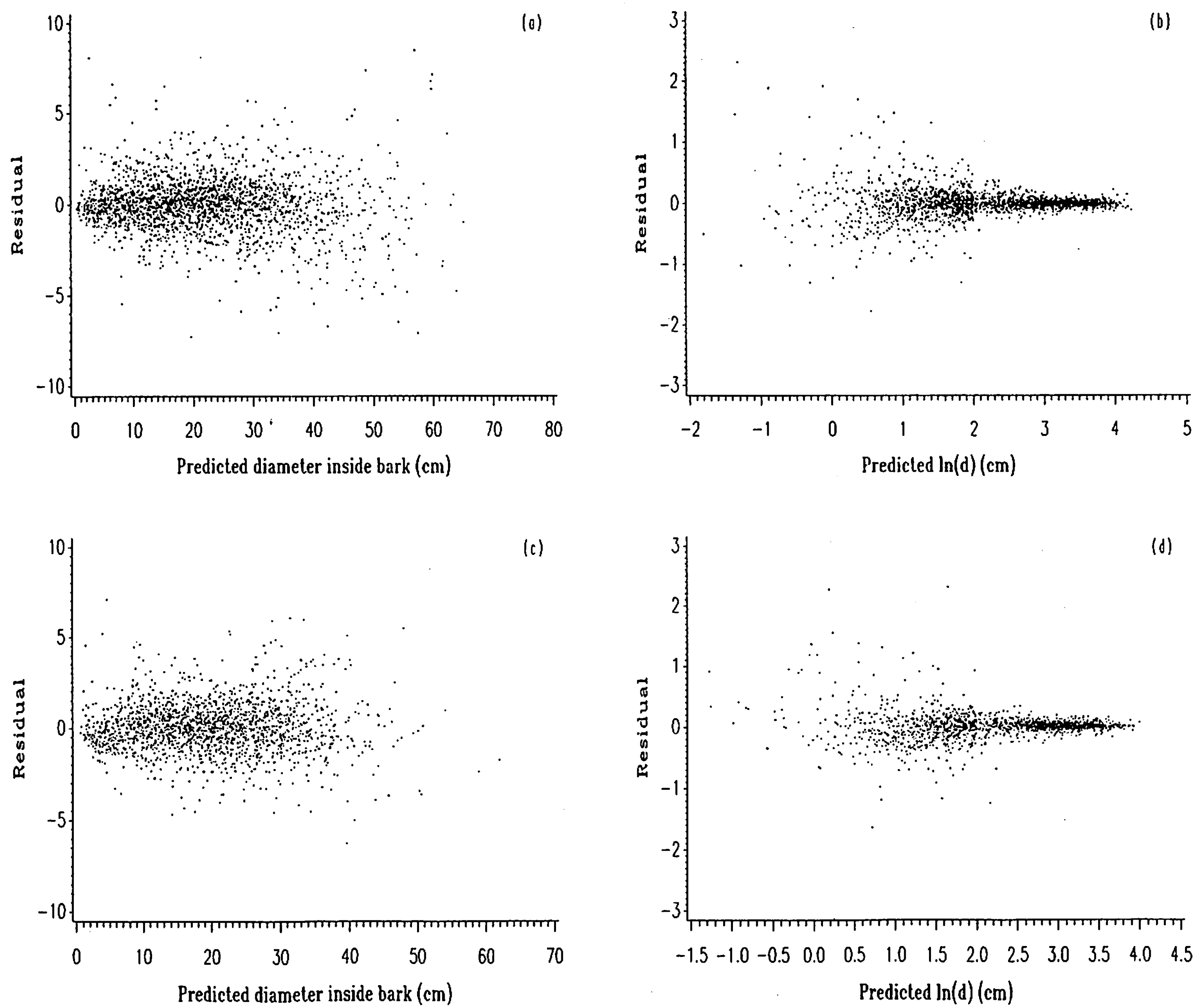

Figure A3. Comparison of residual plots from [A11] and [A10] for white spruce and lodgepole pine (30% of the data for each species are used for plotting): (a) plot for white spruce from [A11]; (b) plot for white spruce from [A10]; (c) plot for lodgepole pine from [A11]; (d) plot for lodgepole pine from [A10].

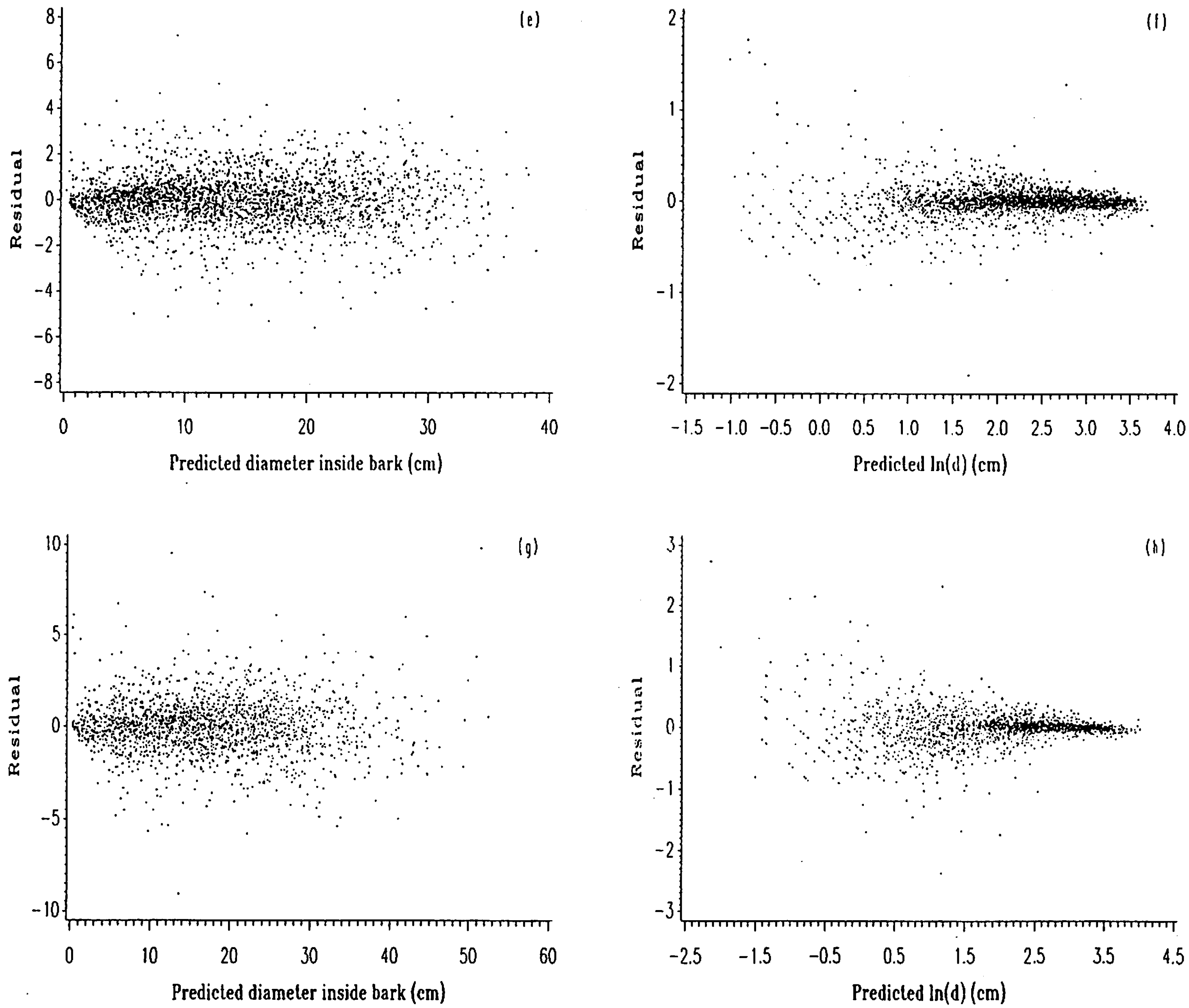

Figure A4. Comparison of residual plots from [A11] and [A10] for jack pine and aspen (100% of the data for jack pine and 30% of the data for aspen are used for plotting): (e) plot for jack pine from [A11]; (f) plot for jack pine from [A10]; (g) plot for aspen from [A11]; (h) plot for aspen from [A10].

equation [A10] for comparison purposes, are shown in Table A1.

In addition to the Douglas-fir data, Kozak's taper equation [A11] was also fitted for all other tree species in Alberta, using the tree sectioning data from the large, provincial stem analysis data base. Starting values of the parameters in nonlinear least squares estimation were obtained from OLS estimation of the linearized equation [A10]. Results of fit statistics for various species are shown in Appendix 2. Using data from the Lower Foothills natural region, four typical residual plots for white spruce (*Picea glauca* (Moench) Voss) and lodgepole pine (*Pinus contorta* var. *latifolia* Engelm.), jack pine (*Pinus banksiana* Lamb.) and aspen (*Populus tremuloides* Michx.) from [A11], along with those comparable residual plots from [A10], are displayed in Figures A3 and A4. Once again, it is clear that the plots show a consistent pattern for these species, implying that the error specification in [A11] is correct, and in [A10] is not. Similar plots were observed for all other species: tamarack (*Larix laricina* (Du Roi) K. Koch), Engelmann spruce (*Picea engelmannii* Parry ex Engelm.), white birch (*Betula papyrifera* Marsh.), balsam poplar (*Populus balsamifera* L.), black spruce (*Picea mariana* (Mill.) B.S.P.), and balsam fir (*Abies balsamea* (L.) Mill.).

Using the procedures described by Gallant (1987), and if the dependence of the data is ignored, significance of the asymptotic *t*-statistics of the parameters can be tested, and different parsimonious versions of the Kozak's nonlinear taper equation for different species can be formulated by dropping parameters or variables that contribute little to the quality of the fit. Perez et al. (1990) demonstrated such analysis for the linearized version of the model. Finding a parsimonious version of the nonlinear form involves fitting nonlinear versions of the taper equation, and ranking them by residual plots and the fit statistics such as R^2, MSE, B and t-statistics of the parameters. However, such analysis was not pursued further here since it would probably be a digression from the main purpose of this analysis. A consistent model form for several tree species is sometimes desirable in applications, and was requested by Alberta Land and Forest Services. In the case of high parameter correlation and non-convergence, where the nonlinear solution is not found, parsimonious versions of the taper equation may be required.

Merchantable Height Calculation

The merchantable height calculation formula for a given top diameter inside bark requires an iterative procedure on the following equation derived from [A11]:

$$[A20] \qquad h_i/H = \left[1-\left(\frac{d_i}{k}\right)^{1/c}(1-\sqrt{p})\right]^2$$

where k and c are defined in equations [A6] and [A7], and the coefficients in [A6] and [A7] are those obtained from nonlinear least squares fit of [A11]. Equation [A20] is appropriate only when the error structure follows that of [A11]. If the linearized equation [A10] is appropriate, the corresponding merchantable height calculation formula should be adjusted with the biasing factor accounted for, that is

$$[A21] \qquad h_i/H = \left[1-\left(\frac{d_i}{ke^{\sigma^2/2}}\right)^{1/c}(1-\sqrt{p})\right]^2$$

where k and c are defined in equations [A6] and [A7], and the coefficients as well as the estimated error variance in this case are those obtained from linear least squares fit of [A10]. It is not difficult to show that equation [A21] can be derived from [A19].

CONCLUSIONS

Kozak's taper model provides a promising tool for accurate individual tree volume predictions in Alberta. As it gains more prominent and widespread application, a certain number of cautionary notes on fitting of the model should be recognized. They relate to some fundamental regression concepts that may be inadvertently ignored in model fitting. The correct method for estimating Kozak's taper equation may be summarized in two steps:

1. Estimate the regression coefficients of equation [A10] by multiple linear least squares methods. Examine the residual plot. If the distribution of values in the plot is satisfactory, equation [A10] is appropriate. Diameters inside bark should be predicted using equation [A19]. Merchantable height

calculation should follow [A21].

2. If the residual plot from [A10] is not satisfactory, estimate equation [A11] using a nonlinear least squares procedure. Examine the residual plot. If satisfactory, equation [A11] is appropriate. Diameter inside bark is directly predicted from [A11]. Merchantable height calculation follows [A20].

The stem analysis data for all major tree species in Alberta suggested that model [A11] is the most appropriate. It is possible that the error structure for Kozak's taper equation is more complicated than those specified in [A11] and [A12]. If this degree of complexity is confirmed, statistical methodologies for weighted regressions described in Carroll and Ruppert (1988), Gallant (1987), and Judge et al. (1988) can be explored.

Traditionally, linearizations were frequently used to linearize nonlinear models so that multiple linear regression methods could be applied. Linearizations were also common as remedial measures for unequal error variances, as well as for nonnormality and dependence of the error terms. However, as demonstrated in this analysis, a difficulty with linearization is that it may create inappropriate regression problems that are largely caused by inappropriate specification of the error structure of the original model. Linearization for the purpose of simplicity in model fitting in lieu of estimating them in their original forms must be implemented with caution.

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Appendix 2.

Natural Region Based Coefficients for Individual Tree Volume Estimation

This appendix provides estimated coefficients and associated fit statistics for the taper model, the diameter outside/inside bark model, the height-diameter model, and the stump diameter and breast height diameter model.

1. Coefficients for the taper model

The taper equation fitted is:

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

where

$$X = (1-\sqrt{h/H})\,/\,(1-\sqrt{p})$$

and

d = diameter inside bark at h (cm)

h = height above the ground (m), $0 \leq h \leq H$

H = total tree height (m)

D = diameter at breast height outside bark (cm)

$Z = h\,/\,H$

p = location of the inflection point, assumed to be at 22.5% of total height above the ground

e = base of the natural logarithm ($\approx$2.71828)

a_0, a_1, a_2, b_1, b_2, b_3, b_4, b_5 = parameters to be estimated.

The coefficients of determination (R^2) and the mean squared errors (MSE) are computed by

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

and

$$MSE = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n-m}$$

where: y_i = observed diameter inside bark, $\hat{y}_i$ = predicted diameter inside bark, $\bar{y}_i$ = observed average diameter inside bark, n = number of observations, and m = number of parameters (m = 8).

Table A2. Coefficients for the taper model for softwood groups

Estimates	Natural region[a]			
	2, 15, 16	9, 11, 14	7, 8, 10	1, 3, 4, 5, 6, 12, 13
a_0	0.858012	0.864073	0.836332	0.907541
a_1	0.994667	1.000696	1.023299	0.972889
a_2	0.998503	0.998194	0.996897	0.999056
b_1	0.957817	1.089652	1.142097	0.838891
b_2	-0.228150	-0.224349	-0.253295	-0.227784
b_3	1.696453	1.584261	1.834277	1.620364
b_4	-0.788021	-0.813796	-0.914346	-0.686296
b_5	0.142355	0.165997	0.121166	0.065843
p	0.225	0.225	0.225	0.225
n	795	17822	15685	13685
R^2	0.9859	0.9813	0.9739	0.9819
MSE	1.1262	2.2872	2.5324	1.6027

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

[a] See Appendix 4 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table A3. Coefficients for the taper model for hardwood groups

Estimates	Natural region[a]			
	2, 14, 15, 16	9, 11	7, 8, 10	1, 3, 4, 5, 6, 12, 13
a_0	0.986975	0.875806	0.553873	0.850133
a_1	0.908801	0.974791	1.182243	0.991087
a_2	1.003121	0.999886	0.991753	0.998750
b_1	0.628126	0.531879	0.600794	0.631153
b_2	-0.061440	-0.049690	-0.058390	-0.085234
b_3	-0.034635	-0.290443	-0.222472	-0.067347
b_4	0.049512	0.184209	0.113434	0.082414
b_5	0.105204	0.073231	0.117909	0.039234
p	0.225	0.225	0.225	0.225
n	3300	9019	2214	17216
R^2	0.9801	0.9794	0.9771	0.9775
MSE	1.2378	1.7739	2.1076	1.8400

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

[a] See Appendix 4 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table A4. Coefficients for the taper model for black spruce

Estimates	Natural region[a]		
	7, 8, 9, 10, 11	1, 2, 3, 4, 5, 6, 12, 13, 14, 15, 16	Provincial
a_0	0.957624	0.929037	0.940695
a_1	0.946740	0.967718	0.957211
a_2	1.000452	0.998511	0.999640
b_1	1.430462	1.236597	1.395784
b_2	-0.356702	-0.308204	-0.344672
b_3	2.950725	2.535507	2.835917
b_4	-1.455471	-1.222060	-1.396460
b_5	0.154263	0.146243	0.152487
p	0.225	0.225	0.225
n	2829	2894	5723
R^2	0.9803	0.9804	0.9807
MSE	0.8314	0.6239	0.7315

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

[a] See Appendix 4 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table A5. Coefficients for the taper model for balsam fir

Estimates	Natural region[a]		
	1 to 6, 9, 11 to 16	7, 8, 10	Provincial
a_0	0.918647	1.108006	1.002016
a_1	0.990225	0.898380	0.944076
a_2	0.997292	1.001816	0.999921
b_1	1.568514	1.338336	1.336330
b_2	-0.384262	-0.304630	-0.320352
b_3	3.503466	2.694363	2.839497
b_4	-1.677185	-1.277617	-1.324815
b_5	0.128169	0.087438	0.077452
p	0.225	0.225	0.225
n	1096	2016	3112
R^2	0.9839	0.9792	0.9803
MSE	1.0419	1.4907	1.4176

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

[a] See Appendix 4 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table A6. Coefficients for the taper model for aspen

Estimates	Natural region[a]				
	2, 14, 15, 16	9, 11	7, 8, 10	1, 3, 4, 5, 6, 12, 13	Provincial
a_0	0.944522	0.905615	0.588838	0.841897	0.790406
a_1	0.938030	0.964894	1.161895	0.997064	1.026943
a_2	1.001644	1.000054	0.992096	0.998713	0.997524
b_1	0.695363	0.553236	0.709300	0.536865	0.600584
b_2	-0.067849	-0.049737	-0.075446	-0.064020	-0.065681
b_3	0.050603	-0.280768	-0.116041	-0.234471	-0.173812
b_4	-0.016330	0.170687	0.040949	0.179963	0.121363
b_5	0.116432	0.075789	0.113638	0.031550	0.063253
p	0.225	0.225	0.225	0.225	0.225
n	2475	7932	2474	14968	27848
R^2	0.9848	0.9804	0.9791	0.9806	0.9804
MSE	0.9788	1.7208	1.9219	1.5698	1.6312

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

[a] See Appendix 4 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table A7. Coefficients for the taper model for balsam poplar

Estimates	Natural region[a]		
	7, 8, 9, 10, 11, 14	1 to 6, 12, 13, 15, 16	Provincial
a_0	0.913329	0.804370	0.861179
a_1	0.922590	0.982874	0.951483
a_2	1.002574	0.999527	1.000957
b_1	0.308448	0.996958	0.752581
b_2	-0.065670	-0.223248	-0.167305
b_3	-0.102130	1.106731	0.693611
b_4	0.226336	-0.459817	-0.224137
b_5	0.023148	-0.003392	0.008214
p	0.225	0.225	0.225
n	1680	1790	3470
R^2	0.9817	0.9710	0.9751
MSE	1.1578	2.6792	1.9692

$$d = a_0 D^{a_1} a_2^D X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3 \sqrt{Z} + b_4 e^Z + b_5 (D/H)}$$

[a] See Appendix 4 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table A8. Coefficients for the taper model for white spruce

Estimates	Natural region[a]			
	7, 8, 10	1 to 6, 12, 13, 15, 16	9, 11, 14	Provincial
a_0	0.713393	0.903528	0.862685	0.860438
a_1	1.071533	0.975136	0.993148	0.995406
a_2	0.996067	0.999018	0.998773	0.998493
b_1	1.153679	0.846981	1.135018	1.040218
b_2	-0.283807	-0.244969	-0.252377	-0.252387
b_3	2.022713	1.783097	1.885321	1.842818
b_4	-0.953783	-0.730236	-0.921437	-0.852227
b_5	0.101608	0.040997	0.150228	0.110359
p	0.225	0.225	0.225	0.225
n	2853	7945	10005	20803
R^2	0.9767	0.9825	0.9852	0.9831
MSE	4.0306	1.8197	2.1530	2.3370

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

[a] See Appendix 4 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table A9. Coefficients for the taper model for lodgepole pine

Estimates	Natural region[a]				
	7, 8	6, 9, 11, 14	4, 10	1, 2, 3, 5, 12, 13, 15, 16	Provincial
a_0	0.800648	0.957164	0.828665	1.033572	0.897617
a_1	1.053544	0.959992	1.024196	0.913621	0.988518
a_2	0.995568	0.999774	0.997492	1.000765	0.998735
b_1	0.568347	0.766747	0.596193	0.256633	0.675759
b_2	-0.125114	-0.140758	-0.118777	-0.049091	-0.130313
b_3	0.610085	0.666037	0.465591	-0.252118	0.570634
b_4	-0.238442	-0.355050	-0.196176	0.174267	-0.275457
b_5	0.045398	0.132140	0.083094	0.123722	0.105403
p	0.225	0.225	0.225	0.225	0.225
n	2042	7656	7376	743	17808
R^2	0.9733	0.9830	0.9817	0.9840	0.9823
MSE	1.1281	1.5764	1.3475	1.0140	1.4503

$$d = a_0 D^{a_1} a_2^D X^{b_1 z^2 + b_2 \ln(z+0.001) + b_3\sqrt{z} + b_4 e^z + b_5 (D/H)}$$

[a] See Appendix 4 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table A10. Coefficients for the taper model for jack pine and other tree species

Estimates	Species				
	Jack pine	Douglas-fir	White birch	Tamarack	Engelmann spruce
a_0	0.940832	0.913153	0.894358	0.933517	1.072576
a_1	0.955575	0.964386	1.007721	0.965471	0.897766
a_2	0.999333	0.998391	0.991384	0.998393	1.001919
b_1	0.116311	1.386315	-0.483072	2.079455	1.301834
b_2	-0.028172	-0.286495	0.155593	-0.462028	-0.305439
b_3	-0.384427	1.783899	-2.273122	3.732057	2.265717
b_4	0.304055	-0.916932	1.326501	-1.950194	-1.119671
b_5	0.072192	0.058830	0.168897	0.190425	0.123519
p	0.225	0.225	0.225	0.225	0.225
n	3562	638	416	225	847
R^2	0.9828	0.9868	0.9828	0.9824	0.9790
MSE	1.0921	1.1199	0.4305	0.8189	2.5719

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 z^2 + b_2 \ln(z+0.001) + b_3\sqrt{z} + b_4 e^{z} + b_5 (D/H)}$$

2. Coefficients for the diameter outside/inside bark model

The diameter outside/inside bark model fitted is:

$$DOB = a + bDIB$$

where:

DOB = diameter outside bark at any point on the stem (cm)

DIB = corresponding diameter inside bark on the stem (cm)

a, b = parameters to be estimated.

The coefficients of determination (R^2) and the mean squared errors (MSE) are computed by

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

and

$$MSE = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n-m}$$

where: y_i = observed diameter outside bark at any point on the stem

$\hat{y}_i$ = predicted diameter outside bark

$\bar{y}_i$ = observed average diameter outside bark

n = number of observations

m = number of parameters (m = 2).

Table A11. Coefficients for the diameter outside/inside bark model

Species	Natural regions[1]	Estimated coefficients a	Estimated coefficients b	n	R^2	MSE
Softwood group	2, 15, 16	0.496341	1.025974	795	0.9984	0.1316
	9, 11, 14	0.447477	1.024646	17196	0.9986	0.1836
	7, 8, 10	0.337955	1.029211	15685	0.9986	0.1482
	1, 3, 4, 5, 6, 12, 13	0.355429	1.031969	13685	0.9987	0.1224
Hardwood group	2, 14, 15, 16	0.052537	1.084738	3300	0.9969	0.2261
	9, 11	0.161950	1.073830	8425	0.9972	0.2918
	7, 8, 10	0.262986	1.072787	2214	0.9974	0.2788
	1, 3, 4, 5, 6, 12, 13	0.052846	1.085439	17220	0.9964	0.3451
Aspen	2, 14, 15, 16	0.024127	1.081189	2476	0.9976	0.1819
	9, 11	0.135261	1.072734	7612	0.9976	0.2444
	7, 8, 10	0.211340	1.073573	2474	0.9977	0.2471
	1, 3, 4, 5, 6, 12, 13	0.061755	1.079512	15016	0.9974	0.2407
	Provincial	0.091433	1.077082	27582	0.9976	0.2397
Balsam/alpine fir	7, 8, 10	0.323616	1.050716	2016	0.9975	0.2007
	1 to 6, 9, 11 to 16	0.249035	1.050236	1096	0.9983	0.1228
	Provincial	0.289940	1.051003	3125	0.9979	0.1751
Balsam poplar	7, 8, 9, 10, 11, 14	0.257085	1.103150	1414	0.9952	0.3947
	1 to 6, 12, 13, 15, 16	0.109731	1.125120	1790	0.9947	0.6157
	Provincial	0.149322	1.117988	3204	0.9948	0.5326
Lodgepole pine	7, 8	0.240347	1.020105	2046	0.9991	0.0416
	6, 9, 11, 14	0.294015	1.024582	7307	0.9991	0.0903
	4, 10	0.308258	1.024549	7407	0.9990	0.0771
	1, 2, 3, 5, 12, 13, 15, 16	0.189744	1.046810	743	0.9963	0.2572
	Provincial	0.283173	1.025305	17608	0.9990	0.0905
Black spruce	7, 8, 9, 10, 11	0.414614	1.030781	2829	0.9985	0.0663
	1 to 6, 12, 13, 14, 15, 16	0.349765	1.036689	2894	0.9982	0.0622
	Provincial	0.382746	1.033405	5723	0.9984	0.0646
White spruce	9, 11, 14	0.536767	1.022700	9681	0.9993	0.1122
	7, 8, 10	0.521645	1.024172	2853	0.9994	0.1161
	1 to 6, 12, 13, 15, 16	0.413577	1.028342	7955	0.9992	0.0851
	Provincial	0.484768	1.024893	20473	0.9993	0.1033
White birch	Provincial	0.077197	1.062798	416	0.9976	0.0666
Douglas-fir	Provincial	-0.095253	1.123153	661	0.9945	0.5852
Tamarack	Provincial	0.378870	1.034008	225	0.9984	0.0795
Jack pine	Provincial	0.161727	1.045672	3563	0.9949	0.3570
Engelmann spruce	Provincial	0.461270	1.023752	848	0.9993	0.0898

$DOB = a + bDIB$

[1] See Appendix 4 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

3. **Coefficients for the height-diameter model**

The height-diameter model fitted is:

$$H = 1.3 + a(1 - e^{-bD})^{c}$$

where:

H = total tree height (m)

D = diameter at breast height outside bark (cm)

e = base of the natural logarithm (≈2.71828)

a, b, c = parameters to be estimated.

The coefficients of determination (R^2) and the mean squared errors (MSE) are computed by

$$R^2 = 1 - \frac{\sum_{i=1}^{n} w_i (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} w_i (y_i - \bar{y})^2}$$

and

$$MSE = \frac{\sum_{i=1}^{n} w_i (y_i - \hat{y}_i)^2}{n - m}$$

where:

y_i = actual tree height

$\hat{y}_i$ = predicted tree height

$\bar{y}_i$ = observed average tree height

n = number of observations

m = number of parameters (m = 3)

$w_i = 1/D_i$.

Table A12. Coefficients for the provincial height-diameter model

Species	Estimated coefficients			n	R^2	MSE
	a	b	c			
Aspen	25.6614	0.06834	1.1394	3604	0.8734	0.3083
White birch	27.9727	0.03522	0.8695	101	0.8565	0.3301
Balsam/alpine fir	24.7532	0.06615	1.5695	497	0.9316	0.2662
Douglas-fir	21.3299	0.06090	1.5973	78	0.7912	0.1679
Tamarack	26.3266	0.05375	1.4026	39	0.8651	0.4101
Balsam poplar	25.5700	0.05050	0.9865	528	0.8067	0.3219
Jack pine	31.4263	0.03888	1.1279	589	0.9181	0.2669
Lodgepole pine	29.0075	0.04859	1.1782	3096	0.7873	0.3599
Black spruce	24.5751	0.05432	1.2243	1570	0.8647	0.2468
Engelmann spruce	36.3184	0.02604	1.0930	153	0.7732	0.3271
White spruce	32.1261	0.04633	1.3032	2889	0.8762	0.4214

$$H = 1.3 + a(1 - e^{-bD})^{c}$$

Table A13. Coefficients for the natural region based height-diameter model

Species	Natural regions[1]	Estimated coefficients a	b	c	n	R^2	MSE
Softwood group	2, 15, 16	30.7738	0.06562	1.6975	89	0.8831	0.3858
	9, 11, 14	32.4540	0.04648	1.3224	2828	0.8187	0.3905
	7, 8, 10	28.4311	0.04513	1.1839	3399	0.9126	0.3049
	1, 3, 4, 5, 6, 12, 13	31.9247	0.04372	1.2310	2594	0.8155	0.3722
Hardwood group	2, 14, 15, 16	27.1014	0.05186	0.9954	410	0.8155	0.3722
	9, 11	25.8069	0.06818	1.2063	1320	0.8491	0.3111
	7, 8, 10	27.7784	0.05235	1.3156	363	0.7094	0.3981
	1, 3, 4, 5, 6, 12, 13	24.6591	0.07797	1.2017	2140	0.9043	0.2697
Aspen	2, 14, 15, 16 1	26.5484	0.05699	0.9846	300	0.8688	0.2755
	9, 11 2	25.6731	0.07367	1.2608	1100	0.8701	0.2877
	7, 8, 10 3	28.0750	0.04860	1.2173	386	0.7073	0.4187
	1, 3, 4, 5, 6, 12, 13 4	24.8408	0.08081	1.2405	1836	0.9136	0.2400
Balsam/alpine fir	7, 8, 10 2	24.3383	0.06707	1.5909	252	0.9570	0.1798
	1 to 6, 9, 11 to 16 1	28.6319	0.05226	1.4467	161	0.9118	0.3496
Balsam poplar	7, 8, 9, 10, 11, 14 1	25.1413	0.06488	1.3192	206	0.7143	0.3361
	1 to 6, 12, 13, 15, 16 2	25.3810	0.05010	0.9270	236	0.8747	0.2840
Lodgepole pine	7, 8 1	24.4114	0.03555	0.7846	320	0.5534	0.2690
	6, 9, 11, 14 2	29.6276	0.05461	1.2997	1080	0.8217	0.2860
	4, 10 3	24.8398	0.06468	1.2937	1602	0.7708	0.3666
	1, 2, 3, 5, 12, 13, 15, 16 4	23.9518	0.07865	1.4813	94	0.8712	0.2302
Black spruce	7, 8, 9, 10, 11 1	24.9305	0.05281	1.2552	1037	0.8660	0.2465
	1 to 6, 12, 13, 14, 15, 16 2	24.3666	0.05775	1.2313	617	0.8737	0.2372
White spruce	9, 11, 14 3	32.4278	0.05055	1.3940	1185	0.8801	0.3681
	7, 8, 10 1	38.3117	0.02635	1.1152	526	0.8614	0.4580
	1 to 6, 12, 13, 15, 16 2	29.8812	0.05557	1.3911	1176	0.9020	0.3339

$H = 1.3 + a(1-e^{-bD})^c$

[1] See Appendix 4 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

4. Coefficients for the stump diameter and breast height diameter model

The stump diameter and breast height diameter model fitted is:

$$DOB_{stp} = a + bD + cD^2$$

where:

DOB_{stp} = stump diameter outside bark (cm) at stump height of 0.3 m

D = diameter at breast height outside bark (cm)

a, b, c = parameters to be estimated.

The coefficients of determination (R^2) and the mean squared errors (*MSE*) are computed by

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

and

$$MSE = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n - m}$$

where:

y_i = actual stump diameter outside bark

$\hat{y}_i$ = predicted stump diameter outside bark

$\bar{y}_i$ = observed average stump diameter outside bark

n = number of observations

m = number of parameters (m = 3).

Table A14. Coefficients for the stump diameter and breast height diameter model

Species	Natural region[1]	Estimated coefficients a	b	c	n	R^2	MSE
Softwood group	2, 15, 16	1.065686	1.006206	0.002586	85	0.9811	1.9421
	9, 11, 14	0.499206	1.044508	0.002113	1752	0.9735	4.4462
	7, 8, 10	-0.710193	1.127362	0.000588	1874	0.9742	3.5347
	1, 3, 4, 5, 6, 12, 13	-0.233569	1.119301	0.000390	1704	0.9828	2.4142
Hardwood group	2, 14, 15, 16	0.487002	0.990354	0.003679	409	0.9840	1.4813
	9, 11	0.095438	1.068140	0.001468	984	0.9753	3.0927
	7, 8, 10	0.007333	1.091470	0.001121	251	0.9716	3.0350
	1, 3, 4, 5, 6, 12, 13	0.047136	1.072125	0.001866	1967	0.9774	3.1221
Aspen	2, 14, 15, 16	0.114041	1.016604	0.003272	300	0.9852	1.4862
	9, 11	-0.093612	1.083466	0.001270	815	0.9762	3.2255
	7, 8, 10	-1.646520	1.239705	-0.001720	225	0.9753	3.2291
	1, 3, 4, 5, 6, 12, 13	-0.052028	1.076457	0.001805	1691	0.9774	3.0374
	Provincial	-0.330572	1.106639	0.000986	3099	0.9780	2.9946
Balsam/alpine fir	7, 8, 10	1.265808	0.952207	0.003527	252	0.9756	2.2304
	1 to 6, 9, 11 to 16	0.664028	1.019720	0.002754	161	0.9833	1.6215
	Provincial	1.028869	0.979696	0.003078	413	0.9802	2.0308
Balsam poplar	7, 8, 9, 10, 11, 14	0.587071	1.070632	0.000464	176	0.9756	2.0649
	1 to 6, 12, 13, 15, 16	0.762353	1.017031	0.002918	211	0.9754	3.8825
	Provincial	0.671062	1.037718	0.002077	387	0.9749	3.1733
Lodgepole pine	7, 8	-0.159741	1.046160	0.001804	280	0.9711	1.2603
	6, 9, 11, 14	0.582463	1.034249	0.001970	706	0.9711	2.9597
	4, 10	-0.487166	1.112282	0.000347	831	0.9753	2.0865
	1, 2, 3, 5, 12, 13, 15, 16	-0.321525	1.175896	-0.001650	91	0.9809	1.7527
	Provincial	-0.245285	1.088419	0.000993	1929	0.9762	2.3222
Black spruce	7, 8, 9, 10, 11	0.982793	0.943279	0.006344	439	0.9676	1.8523
	1 to 6, 12, 13, 14, 15, 16	0.125198	1.058588	0.002137	481	0.9767	0.9909
	Provincial	0.536125	0.996309	0.004617	920	0.9725	1.4253
White spruce	9, 11, 14	0.190577	1.069563	0.001822	892	0.9700	6.0680
	7, 8, 10	-2.112193	1.275528	-0.001564	298	0.9677	8.6912
	1 to 6, 12, 13, 15, 16	-0.582516	1.156757	-0.000271	879	0.9806	3.1438
	Provincial	-0.567783	1.142153	0.000429	2069	0.9750	5.3164
White birch	Provincial	0.300399	1.157729	-0.001896	71	0.9760	0.7442
Douglas-fir	Provincial	-1.689559	1.380771	-0.004699	80	0.9738	2.8534
Tamarack	Provincial	0.499971	1.065686	0.002899	34	0.9868	0.9720
Jack pine	Provincial	0.211020	1.144627	-0.001614	519	0.9794	2.1163
Engelmann spruce	Provincial	-0.107514	1.101339	0.001312	107	0.9732	4.6559

$$DOB_{stp} = a+bD+cD^2$$

[1] See Appendix 4 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Appendix 3.

An Example Program for Calculating Tree Volumes

This Statistical Analysis System (SAS) program shows step-by-step computations for merchantable length, gross merchantable volume, gross total volume, trees/m^3 merchantable volume, total tree height, and stump diameter. It also places the calculated results into formats that can be easily modified into desired tables. Actual programs used for all calculations are available upon request.

```
*This program calculates total tree volume, merchantable length/volume, trees/m³ merchantable volume,
  total tree height, and stump diameter;
*Read the height and diameter data, in this case, a set of trees with dbh from 2 cm to 80 cm by 2 cm
  classes, total tree height (ht) from 4 m to 40 m by 2 m classes were generated;
*Make sure dbh ≥ minimum top diameter inside bark;

data v1;
   do dbh=2 to 80 by 2;
     do ht=4 to 40 by 2; output;
     end;
  end;
run;

*The iteration procedure, using estimated coefficients for softwood from natural regions 2, 15, and 16;

data v2;
  set v1;
        a0 = 0.858012;      a1 = 0.994667;      a2 = 0.998503;      b1 = 0.957817;
        b2 = -0.228150;     b3 = 1.696453;      b4 = -0.788021;     b5 = 0.142355;

*Define g = h/ht, set the initial value for g;

        g0 = 0.9;

*The following iteration process is repeated until the desired precision is obtained;
*A 2.0 cm top diameter inside bark is assumed;

        do until(abs(g0-g1) < 0.00000001);
         c = b1*(g0)**2 + b2*log(g0+0.001) + b3*sqrt(g0) + b4*exp(g0) + b5*(dbh/ht);
         g1 = (1 - (( 2/(a0*dbh**a1*a2**dbh))**(1/c))*(1 - sqrt(0.225)))**2;
         g0 = (g0 + g1)/2;
        end;

*Keep the coefficients and the final g0 or g1;

        keep a0-a2 b1-b5 dbh ht g0 g1;
run;

data v3;
   set v2;

*Compute merchantable height (hi) and merchantable length (mlen);
*A stump height of 0.30 m is assumed;

   hi = g0*ht;
   mlen = hi - 0.3;

*Divide merchantable length into 10 sections of equal length;
*Compute the height above the ground from the middle and the top of each section;
```

```
mlen1 = 1*(hi - 0.3)/20 + 0.3;      mlen11 = 11*(hi - 0.3)/20 + 0.3;
mlen2 = 2*(hi - 0.3)/20 + 0.3;      mlen12 = 12*(hi - 0.3)/20 + 0.3;
mlen3 = 3*(hi - 0.3)/20 + 0.3;      mlen13 = 13*(hi - 0.3)/20 + 0.3;
mlen4 = 4*(hi - 0.3)/20 + 0.3;      mlen14 = 14*(hi - 0.3)/20 + 0.3;
mlen5 = 5*(hi - 0.3)/20 + 0.3;      mlen15 = 15*(hi - 0.3)/20 + 0.3;
mlen6 = 6*(hi - 0.3)/20 + 0.3;      mlen16 = 16*(hi - 0.3)/20 + 0.3;
mlen7 = 7*(hi - 0.3)/20 + 0.3;      mlen17 = 17*(hi - 0.3)/20 + 0.3;
mlen8 = 8*(hi - 0.3)/20 + 0.3;      mlen18 = 18*(hi - 0.3)/20 + 0.3;
mlen9 = 9*(hi - 0.3)/20 + 0.3;      mlen19 = 19*(hi - 0.3)/20 + 0.3;
mlen10= 10*(hi - 0.3)/20 + 0.3;     mlen20 = 20*(hi - 0.3)/20 + 0.3;

*Prediction of diameter inside bark at the middle and top of each section, using the taper equation;
*Diameter inside bark at stump height is also predicted with stump height = 0.3 metres;

z1 = mlen1 / ht;           z11 = mlen11/ ht;
z2 = mlen2 / ht;           z12 = mlen12/ ht;
z3 = mlen3 / ht;           z13 = mlen13/ ht;
z4 = mlen4 / ht;           z14 = mlen14/ ht;
z5 = mlen5 / ht;           z15 = mlen15/ ht;
z6 = mlen6 / ht;           z16 = mlen16/ ht;
z7 = mlen7 / ht;           z17 = mlen17/ ht;
z8 = mlen8 / ht;           z18 = mlen18/ ht;
z9 = mlen9 / ht;           z19 = mlen19/ ht;
z10 = mlen10/ ht;          z20 = mlen20/ ht;

x1 = (1 - sqrt(z1 )) / (1 - sqrt(.225));      x11 = (1 - sqrt(z11)) / (1 - sqrt(.225));
x2 = (1 - sqrt(z2 )) / (1 - sqrt(.225));      x12 = (1 - sqrt(z12)) / (1 - sqrt(.225));
x3 = (1 - sqrt(z3 )) / (1 - sqrt(.225));      x13 = (1 - sqrt(z13)) / (1 - sqrt(.225));
x4 = (1 - sqrt(z4 )) / (1 - sqrt(.225));      x14 = (1 - sqrt(z14)) / (1 - sqrt(.225));
x5 = (1 - sqrt(z5 )) / (1 - sqrt(.225));      x15 = (1 - sqrt(z15)) / (1 - sqrt(.225));
x6 = (1 - sqrt(z6 )) / (1 - sqrt(.225));      x16 = (1 - sqrt(z16)) / (1 - sqrt(.225));
x7 = (1 - sqrt(z7 )) / (1 - sqrt(.225));      x17 = (1 - sqrt(z17)) / (1 - sqrt(.225));
x8 = (1 - sqrt(z8 )) / (1 - sqrt(.225));      x18 = (1 - sqrt(z18)) / (1 - sqrt(.225));
x9 = (1 - sqrt(z9 )) / (1 - sqrt(.225));      x19 = (1 - sqrt(z19)) / (1 - sqrt(.225));
x10= (1 - sqrt(z10)) / (1 - sqrt(.225));      x20 = (1 - sqrt(z20)) / (1 - sqrt(.225));

dibm0 = (a0*dbh**a1)*(a2**dbh)*
  ((1 - sqrt(0.3/ht)) / (1 - sqrt(.225)))**
  (b1*(0.3/ht)**2+b2*log(0.3/ht+0.001)+b3*sqrt(0.3/ht)
     +b4*exp(0.3/ht)+b5*dbh/ht);

dibm1 = (a0*dbh**a1)*(a2**dbh)*x1**(b1*z1**2+b2*log(z1+0.001)
      +b3*sqrt(z1)+b4*exp(z1)+b5*dbh/ht);
dibm2 = (a0*dbh**a1)*(a2**dbh)*x2**(b1*z2**2+b2*log(z2+0.001)
      +b3*sqrt(z2)+b4*exp(z2)+b5*dbh/ht);
dibm3 = (a0*dbh**a1)*(a2**dbh)*x3**(b1*z3**2+b2*log(z3+0.001)
      +b3*sqrt(z3)+b4*exp(z3)+b5*dbh/ht);
dibm4 = (a0*dbh**a1)*(a2**dbh)*x4**(b1*z4**2+b2*log(z4+0.001)
      +b3*sqrt(z4)+b4*exp(z4)+b5*dbh/ht);
```

```
dibm5 = (a0*dbh**a1)*(a2**dbh)*x5**(b1*z5**2+b2*log(z5+0.001)
      +b3*sqrt(z5)+b4*exp(z5)+b5*dbh/ht);
dibm6 = (a0*dbh**a1)*(a2**dbh)*x6**(b1*z6**2+b2*log(z6+0.001)
      +b3*sqrt(z6)+b4*exp(z6)+b5*dbh/ht);
dibm7 = (a0*dbh**a1)*(a2**dbh)*x7**(b1*z7**2+b2*log(z7+0.001)
      +b3*sqrt(z7)+b4*exp(z7)+b5*dbh/ht);
dibm8 = (a0*dbh**a1)*(a2**dbh)*x8**(b1*z8**2+b2*log(z8+0.001)
      +b3*sqrt(z8)+b4*exp(z8)+b5*dbh/ht);
dibm9 = (a0*dbh**a1)*(a2**dbh)*x9**(b1*z9**2+b2*log(z9+0.001)
      +b3*sqrt(z9)+b4*exp(z9)+b5*dbh/ht);
dibm10 = (a0*dbh**a1)*(a2**dbh)*x10**(b1*z10**2+b2*log(z10+0.001)
      +b3*sqrt(z10)+b4*exp(z10)+b5*dbh/ht);

dibm11 = (a0*dbh**a1)*(a2**dbh)*x11**(b1*z11**2+b2*log(z11+0.001)
      +b3*sqrt(z11)+b4*exp(z11)+b5*dbh/ht);
dibm12 = (a0*dbh**a1)*(a2**dbh)*x12**(b1*z12**2+b2*log(z12+0.001)
      +b3*sqrt(z12)+b4*exp(z12)+b5*dbh/ht);
dibm13 = (a0*dbh**a1)*(a2**dbh)*x13**(b1*z13**2+b2*log(z13+0.001)
      +b3*sqrt(z13)+b4*exp(z13)+b5*dbh/ht);
dibm14 = (a0*dbh**a1)*(a2**dbh)*x14**(b1*z14**2+b2*log(z14+0.001)
      +b3*sqrt(z14)+b4*exp(z14)+b5*dbh/ht);
dibm15 = (a0*dbh**a1)*(a2**dbh)*x15**(b1*z15**2+b2*log(z15+0.001)
      +b3*sqrt(z15)+b4*exp(z15)+b5*dbh/ht);
dibm16 = (a0*dbh**a1)*(a2**dbh)*x16**(b1*z16**2+b2*log(z16+0.001)
      +b3*sqrt(z16)+b4*exp(z16)+b5*dbh/ht);
dibm17 = (a0*dbh**a1)*(a2**dbh)*x17**(b1*z17**2+b2*log(z17+0.001)
      +b3*sqrt(z17)+b4*exp(z17)+b5*dbh/ht);
dibm18 = (a0*dbh**a1)*(a2**dbh)*x18**(b1*z18**2+b2*log(z18+0.001)
      +b3*sqrt(z18)+b4*exp(z18)+b5*dbh/ht);
dibm19 = (a0*dbh**a1)*(a2**dbh)*x19**(b1*z19**2+b2*log(z19+0.001)
      +b3*sqrt(z19)+b4*exp(z19)+b5*dbh/ht);
dibm20 = (a0*dbh**a1)*(a2**dbh)*x20**(b1*z20**2+b2*log(z20+0.001)
      +b3*sqrt(z20)+b4*exp(z20)+b5*dbh/ht);

*Predicted top diameter inside bark dibm20 should be equal to the specified top dib, that is, dibm20 =
 2.0 cm;
*Compute the merchantable volume (mvol) of the tree to 2.0 cm top dib;
*Calculate mvol in terms of cubic metre using Newton's formula;

mvol = 0.00007854*(((hi-0.3)/10)/6)*(dibm0**2 +4*dibm1**2 +dibm2**2) +
       0.00007854*(((hi-0.3)/10)/6)*(dibm2**2 +4*dibm3**2 +dibm4**2) +
       0.00007854*(((hi-0.3)/10)/6)*(dibm4**2 +4*dibm5**2 +dibm6**2) +
       0.00007854*(((hi-0.3)/10)/6)*(dibm6**2 +4*dibm7**2 +dibm8**2) +
       0.00007854*(((hi-0.3)/10)/6)*(dibm8**2 +4*dibm9**2 +dibm10**2) +
       0.00007854*(((hi-0.3)/10)/6)*(dibm10**2+4*dibm11**2+dibm12**2) +
       0.00007854*(((hi-0.3)/10)/6)*(dibm12**2+4*dibm13**2+dibm14**2) +
       0.00007854*(((hi-0.3)/10)/6)*(dibm14**2+4*dibm15**2+dibm16**2) +
       0.00007854*(((hi-0.3)/10)/6)*(dibm16**2+4*dibm17**2+dibm18**2) +
       0.00007854*(((hi-0.3)/10)/6)*(dibm18**2+4*dibm19**2+dibm20**2) ;
```

*Compute trees/m^3 merchantable volume, tip volume, stump volume, and total volume;

```
trees=1/mvol;
tipvol=0.00007854*dibm20**2*(ht-hi)/3;
volstp=0.00007854*dibm0**2*0.3;
tvol = mvol + tipvol + volstp;

keep dbh ht mlen mvol trees tvol;
run;
```

*The above program completes the calculation;
*The following statements are used to arrange the calculated values into different tables;

*Arrange total volume into the table format, only the selected variables are outputted;

```
data v4;
  set v3;
  tv1=lag18(tvol);  tv7=lag12(tvol);  tv13=lag6(tvol);
  tv2=lag17(tvol);  tv8=lag11(tvol);  tv14=lag5(tvol);
  tv3=lag16(tvol);  tv9=lag10(tvol);  tv15=lag4(tvol);
  tv4=lag15(tvol);  tv10=lag9(tvol);  tv16=lag3(tvol);
  tv5=lag14(tvol);  tv11=lag8(tvol);  tv17=lag2(tvol);
  tv6=lag13(tvol);  tv12=lag7(tvol);  tv18=lag1(tvol);

if ht = 40;
```

*Prediction of total tree height, two boundary values of stump diameter;

```
  h = 1.3+30.773756*(1-exp(-0.065620*dbh))**1.697549;
      D1 = DBH - 0.9; D2 = DBH + 1.0;
      STUMP1 = 1.065686 + 1.006206*D1 + 0.002586*D1**2;
      STUMP2 = 1.065686 + 1.006206*D2 + 0.002586*D2**2;
      MX = '-';
run;

data p1;
  file 'a:tsoft2x.dat' lrecl=220;
    set v4;
      PUT D1 1-4 .1 MX $ 5 D2 6-9 .1
      STUMP1 12-15 .1 MX $ 16 STUMP2 17-20 .1
      tv1 21-28 .4          tv2 29-36 .4          tv3 37-44 .4    tv4 45-52 .4
      tv5 53-60 .4          tv6 61-68 .4          tv7 69-76 .4    tv8 77-84 .4
      tv9 85-92 .4          tv10 93-100 .4
      tv11 101-108 .4  tv12 109-116 .4
      tv13 117-124 .4  tv14 125-132 .4
      tv15 133-140 .4  tv16 141-148 .4
      tv17 149-156 .4  tv18 157-164 .4
      tvol 165-172 .4  h 173-180 .1;
run;
```

*Arrange merchantable length/volume into the table format, only the selected variables are outputted;

```
data v5;
 set v3;
 mv1=lag18(mvol);  mv7=lag12(mvol);  mv13=lag6(mvol);
 mv2=lag17(mvol);  mv8=lag11(mvol);  mv14=lag5(mvol);
 mv3=lag16(mvol);  mv9=lag10(mvol);  mv15=lag4(mvol);
 mv4=lag15(mvol);  mv10=lag9(mvol);  mv16=lag3(mvol);
 mv5=lag14(mvol);  mv11=lag8(mvol);  mv17=lag2(mvol);
 mv6=lag13(mvol);  mv12=lag7(mvol);  mv18=lag1(mvol);

 m1=lag18(mlen);   m7=lag12(mlen);   m13=lag6(mlen);
 m2=lag17(mlen);   m8=lag11(mlen);   m14=lag5(mlen);
 m3=lag16(mlen);   m9=lag10(mlen);   m15=lag4(mlen);
 m4=lag15(mlen);   m10=lag9(mlen);   m16=lag3(mlen);
 m5=lag14(mlen);   m11=lag8(mlen);   m17=lag2(mlen);
 m6=lag13(mlen);   m12=lag7(mlen);   m18=lag1(mlen);

 m='/';

 if ht=40;
```

*Prediction of total tree height, two boundary values of stump diameter;

```
  h=1.3+30.773756*(1-exp(-0.065620*dbh))**1.697549;
    D1=DBH-0.9; D2=DBH+1.0;
       STUMP1= 1.065686+1.006206*D1+0.002586*D1**2;
       STUMP2= 1.065686+1.006206*D2+0.002586*D2**2;
       MX='-';
      run;

data p2;
 file 'a:m2s2.dat' lrecl=256;
 set v5;
 PUT D1 1-4 .1 MX $ 5 D2 6-9 .1
     STUMP1 12-15 .1 MX $ 16 STUMP2 17-20 .1
     m1 21-26 .2   m $ 27 mv1 28-33 .4
     m2 34-39 .2  m $ 40 mv2 41-46 .4
     m3 47-52 .2  m $ 53 mv3 54-59 .4
     m4 60-65 .2  m $ 66 mv4 67-72 .4
     m5 73-78 .2  m $ 79 mv5 80-85 .4
     m6 86-91 .2  m $ 92 mv6 93-98 .4
     m7 99-104 .2  m $ 105 mv7 106-111 .4
     m8 112-117 .2  m $ 118 mv8 119-124 .4
     m9 125-130 .2  m $ 131 mv9 132-137 .4
     m10 138-143 .2  m $ 144 mv10 145-150 .4
     m11 151-156 .2  m $ 157 mv11 158-163 .4
     m12 164-169 .2  m $ 170 mv12 171-176 .4
     m13 177-182 .2  m $ 183 mv13 184-189 .4
```

```
    m14 190-195 .2  m $ 196 mv14 197-202 .4
    m15 203-208 .2  m $ 209 mv15 210-215 .4
    m16 216-221 .2  m $ 222 mv16 223-228 .4
    m17 229-234 .2  m $ 235 mv17 236-241 .4
    m18 242-247 .2  m $ 248 mv18 249-254 .4
    mlen 255-260 .2 m $ 261 mvol 262-267 .4
    h 268-275 .1;
 run;

data v6;
  set v3;
 n1=lag18(trees);   n2=lag17(trees);  n3=lag16(trees);  n4=lag15(trees);  n5=lag14(trees);
 n6=lag13(trees);   n7=lag12(trees);  n8=lag11(trees);  n9=lag10(trees);  n10=lag9(trees);
 n11=lag8(trees);   n12=lag7(trees);  n13=lag6(trees);  n14=lag5(trees);  n15=lag4(trees);
 n16=lag3(trees);   n17=lag2(trees);  n18=lag1(trees);

   if ht=40;

*Prediction of total tree height, two boundary values of stump diameter;

  h=1.3+30.773756*(1-exp(-0.065620*dbh))**1.697549;
   D1=DBH-0.9; D2=DBH+1.0;
   STUMP1= 1.065686+1.006206*D1+0.002586*D1**2;
   STUMP2= 1.065686+1.006206*D2+0.002586*D2**2;
   MX='-';
   run;

*Arrange trees/m3 merchantable volume into the table format, only the selected variables are outputed;

data p3;
 file 'a:tr2s2.dat' lrecl=200;
  set v6;
  PUT D1 1-4 .1 MX $ 5 D2 6-9 .1
      STUMP1 12-15 .1 MX $ 16 STUMP2 17-20 .1
      n1 26-33 .3 n2 34-41 .3 n3 42-49 .3 n4 50-57 .3 n5 58-65 .3
      n6 66-73 .3 n7 74-81 .3 n8 82-89 .3 n9 90-97 .3 n10 98-105 .3
      n11 106-113 .3 n12 114-121 .3 n13 122-129 .3
      n14 130-137 .3 n15 138-145 .3
      n16 146-153 .3 n17 154-161 .3 n18 162-169 .3 n19 170-177 .3
      h 178-185 .1;
  run;
```

Appendix 4.

List of Natural Regions of Alberta

Natural region 1 — Central mixedwood

Natural region 2 — Dry mixedwood

Natural region 3 — Wetland mixedwood

Natural region 4 — Sub-Arctic

Natural region 5 — Peace River Lowlands

Natural region 6 — Boreal Highlands

Natural region 7 — Alpine

Natural region 8 — Sub-Alpine

Natural region 9 — Montane

Natural region 10 — Upper Foothills

Natural region 11 — Lower Foothills

Natural region 12 — Athabasca Plain

Natural region 13 — Kazan Upland

Natural region 14 — Foothills Parkland

Natural region 15 — Peace River Parkland

Natural region 16 — Central Parkland

Natural region 17 — Dry mixedgrass

Natural region 18 — Foothills Fescue

Natural region 19 — Northern Fescue

Natural region 20 — Mixedgrass

Appendix 5.

List of Major Alberta Tree Species and Their Species Code

SPECIES	SPECIES CODE	SCIENTIFIC NAME
White spruce	Sw	*Picea glauca* (Moench) Voss
Tamarack	Lt	*Larix laricina* (Du Roi) K. Koch
Engelmann spruce	Se	*Picea engelmannii* Parry ex Engelm.
Lodgepole pine	Pl	*Pinus contorta* var. *latifolia* Engelm.
Jack pine	Pj	*Pinus banksiana* Lamb.
Aspen	Aw	*Populus tremuloides* Michx.
White birch	Bw	*Betula papyrifera* Marsh.
Balsam poplar	Pb	*Populus balsamifera* L.
Black spruce	Sb	*Picea mariana* (Mill.) B.S.P.
Balsam fir	Fb	*Abies balsamea* (L.) Mill.
Alpine fir	Fa	*Abies lasiocarpa* (Hook.) Nutt.
Douglas-fir.	Fd	*Pseudotsuga menziesii* (Mirb.) Franco

Appendix 6.

Metric Conversion Chart

1 cm	= 0.39370 in.
1 m	= 3.28083 ft.
1 ha	= 2.47105 acres
1 m^2	= 10.76385 sq. ft.
1 m^3	= 35.31435 cu. ft
1 km	= 0.62137 miles
1 m^2/ha	= 4.3560 sq. ft/acre
1 m^3/ha	= 14.2913 cu. ft/acre
1 in.	= 2.5400 cm
1 ft.	= 0.3048 m
1 acre	= 0.4047 ha
1 sq. ft.	= 0.09290 m^2
1 cu. ft.	= 0.02832 m^3
1 mile	= 1.6093 km
1 fbm	= 1 ft. $\times$ 1 ft. $\times$ 1 in.
1 Mfbm	= 1000 foot board measure (fbm)
1 m^3 log	$\approx$ 233 board feet lumber (provincial average conversion factor)
1 Mfbm	$\approx$ 4.3 m^3 log (provincial average conversion factor)

Appendix 7.

An Example Program for Fitting the Taper Model

This Statistical Analysis System (SAS) program shows the fitting of Kozak's (1988) taper model on the stem analysis data from Forest Resource Information Branch, Land and Forest Services. The model is fitted by nonlinear least squares, with initial values of the parameters estimated from linear least squares fit of the transformed model. See Appendix 1 for a description of the taper model.

```
*This program shows the fitting of Kozak's (1988) taper model;
*Read the stem analysis data;

data data1;
  infile 'a:\data\treesec.dat';

  retain hag twp rge merid stand plot tree spcode dead dbh ht sec length
          dib1 dob1 dib2 dob2 trcat1 trcat2 trcat3 trcat4 trcat5;

  input retype $ 28 @;
  if retype = 'H' then do;
  hag = 0;
  input vsr 1-2 merid 9 rge 10-11 twp 12-14 stand 15-18 plot 20-25 tree 26-27 spcode $ 33-34
        dead $ 35 secnum 36-37 dbh 38-41 .1 trcat1 44-45 trcat2 46-47 trcat3 48-49
        trcat4 50-51 trcat5 52-53 ht 55-59 .2;
  end;

  if retype = 'S' then do;
  input vsr 1-2 merid 9 rge 10-11 twp 12-14 stand 15-18 plot 20-25 tree 26-27 sec 29-30 length 31-34 .2
        dib1 35-38 .1 dob1 39-42 .1 dib2 43-46 .1 dob2 47-50 .1;

*Calculate height above ground (hag);

    hag = hag + length;

    if dib1 = 0 or dib2 = 0 then delete;
    if dib1 = . or dib2 = . then delete;

*Calculate diameter inside bark (dib) and diameter outside bark (dob);

    dib = (dib1 + dib2) / 2;
    dob = (dob1 + dob2) / 2;

    if dib > 200 then delete;      if dob > 200 then delete;
    if dib > 0 and dob = 0 then delete;
    if dead = 'X' then delete;

*Delete the top section (this step is only needed for the linearized fit);

    if ht = hag then delete;

*Delete trees that are forked or having broken top;

   if trcat1 = 13 or trcat2 = 13 or trcat3 = 13 or trcat4 = 13 or trcat5 = 13 or
        trcat1 = 19 or trcat2 = 19 or trcat3 = 19 or trcat4 = 19 or trcat5 = 19 then delete;

keep vsr hag twp rge merid stand plot tree spcode dbh ht sec length dib dob;
    output;      end;
run;
```

```
*Fit the taper model;

data data2;
   set data1;

  hr = (ht-hag)/ht; if hr >1 or hr < 0 then delete;
  dbhhr = dbh*hr;   if dbhhr = 0 and dib > 0 then delete;

*Find the initial values by linearizing the taper equation;

   lndib = log(dib);
   lndbh = log(dbh);
   x = (1 - sqrt(hag/ht)) / (1 - sqrt(.225));
   z = hag / ht;
/*
   lnx = log(x);
   lnxz2 = lnx * z * z;
   lnz001 = log(z + .001);
   lnxlnz01 = lnx * lnz001;
   lnxsqrz = lnx * sqrt(z);
   lnxez = lnx * exp(z);
   lnxdh = lnx * (dbh / ht);

proc reg data = data2;
   model lndib = lndbh dbh lnxz2 lnxlnz01 lnxsqrz lnxez lnxdh;
   output out = res1 p = pred r = resid;
 proc plot data = res1;
   plot resid*pred;
 run;
*/
*Estimated coefficients from linearized fit are used as initial values in nonlinear estimation;
*The comment portion of the program, /* ...... */, is deleted in the final fitting of the nonlinear taper
  model;

proc nlin method = dud data = data2;
  parms a0 = 0.95 a1 = 0.95 a2 = 1.0020 b1 = 1.60 b2 = -0.32 b3 = 2.53 b4 = -1.2712 b5 = 0.102;

        c = b1*z**2 + b2*log(z + 0.001) + b3*sqrt(z) + b4*exp(z) + b5*dbh/ht;

   model dib = a0*dbh**a1*a2**dbh*x**c;
   output out = res1 p = pred r = resid;
 run;

 proc plot data = res1;
   plot resid*pred;
 run;
```