Ecologically Based Individual Tree Volume Estimation for Major Alberta Tree Species

Report # 3

Summary of Equations and Estimated Coefficients for Ecologically Based Individual Tree Volume Estimation in Alberta

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REPORT SERIES:

ECOLOGICALLY BASED INDIVIDUAL TREE VOLUME ESTIMATION FOR MAJOR ALBERTA TREE SPECIES

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1). Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco)
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ABSTRACT

This report presents models and estimated natural region based coefficients associated with individual tree volume estimation in Alberta. The models include the taper model, the diameter outside and inside bark model, the height-diameter model, and the stump diameter and breast height diameter model. Tables displayed in other publications of this series, Ecologically Based Individual Tree Volume Estimation for Major Alberta Tree Species, were formulated according to the estimated coefficients described herein.

1.0 INTRODUCTION

This report presents summary of models and estimated coefficients for natural region based individual tree volume estimation in Alberta. It includes descriptions for the taper model, the diameter outside and inside bark model, the height-diameter model, and the stump diameter and breast height diameter model. All tables displayed in other publications of this series, Ecologically based Individual Tree Volume Estimation for Major Alberta Tree Species, were formulated according to the estimated coefficients described herein.

A more detailed description of the procedures for fitting of the models and using of the estimated coefficients for constructions of the tables are presented in Report #1 of this series: Individual Tree Volume Estimation Procedures for Alberta: Methods of Formulation and Statistical Foundations.

2.0 THE TAPER MODEL

The taper model takes the form of

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

where:

$$X = (1-\sqrt{h/H}) / (1-\sqrt{p})$$

and

d = diameter inside bark (cm) at h

h = height above the ground (m), $0 \leq h \leq H$

H = total tree height (m)

D = diameter at breast height outside bark (cm)

$Z = h / H$

p = location of the inflection point, assumed to be at 22.5% of total height above the ground

e = base of the natural logarithm (≈ 2.71828)

a_0, a_1, a_2, b_1, b_2, b_3, b_4, b_5 = parameters to be estimated.

The coefficients of determination (R^2) and the mean squared errors (MSE) are computed by

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

and

$$MSE = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n-m}$$

where: y_i = observed diameter inside bark, $\hat{y}_i$ = predicted diameter inside bark, $\bar{y}_i$ = observed average diameter inside bark, n = number of observations, and m = number of parameters (m = 8).

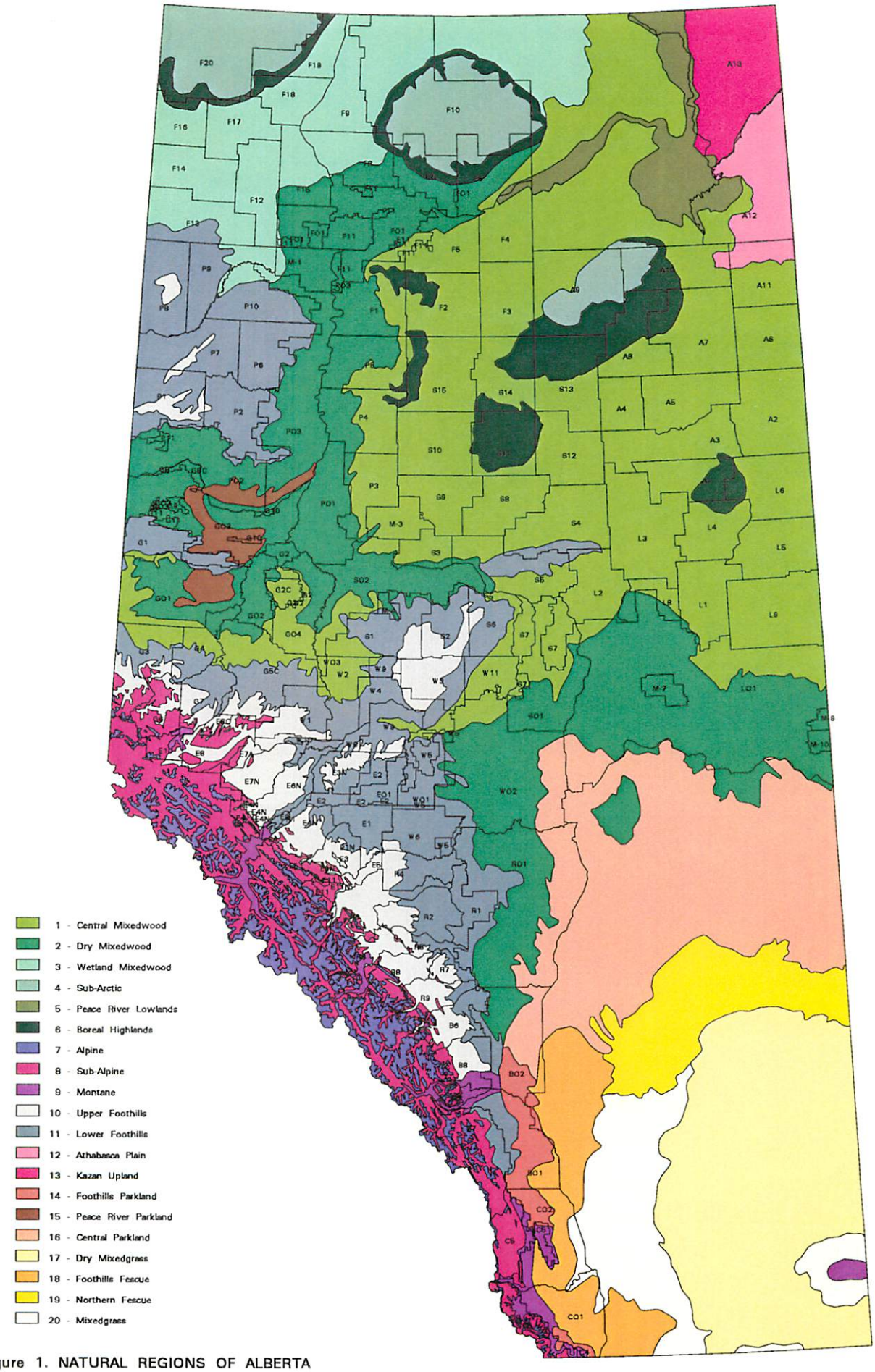

Figure 1. NATURAL REGIONS OF ALBERTA

Produced by Forest Resource Information Branch, Edmonton

Scale 1:5 000 000

Table 1. Coefficients for the taper model for softwood groups

Estimates	Natural region[a]			
	2, 15, 16	9, 11, 14	7, 8, 10	1, 3, 4, 5, 6, 12, 13
a_0	0.858012	0.864073	0.836332	0.907541
a_1	0.994667	1.000696	1.023299	0.972889
a_2	0.998503	0.998194	0.996897	0.999056
b_1	0.957817	1.089652	1.142097	0.838891
b_2	-0.228150	-0.224349	-0.253295	-0.227784
b_3	1.696453	1.584261	1.834277	1.620364
b_4	-0.788021	-0.813796	-0.914346	-0.686296
b_5	0.142355	0.165997	0.121166	0.065843
p	0.225	0.225	0.225	0.225
n	795	17822	15685	13685
R^2	0.9859	0.9813	0.9739	0.9819
MSE	1.1262	2.2872	2.5324	1.6027

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

[a] See Appendix 1 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table 2. Coefficients for the taper model for hardwood groups

Estimates	Natural region[a]			
	2, 14, 15, 16	9, 11	7, 8, 10	1, 3, 4, 5, 6, 12, 13
a_0	0.986975	0.875806	0.553873	0.850133
a_1	0.908801	0.974791	1.182243	0.991087
a_2	1.003121	0.999886	0.991753	0.998750
b_1	0.628126	0.531879	0.600794	0.631153
b_2	-0.061440	-0.049690	-0.058390	-0.085234
b_3	-0.034635	-0.290443	-0.222472	-0.067347
b_4	0.049512	0.184209	0.113434	0.082414
b_5	0.105204	0.073231	0.117909	0.039234
p	0.225	0.225	0.225	0.225
n	3300	9019	2214	17216
R^2	0.9801	0.9794	0.9771	0.9775
MSE	1.2378	1.7739	2.1076	1.8400

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

[a] See Appendix 1 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table 3. Coefficients for the taper model for black spruce

Estimates	Natural region[a]		
	7, 8, 9, 10, 11	1, 2, 3, 4, 5, 6, 12, 13, 14, 15, 16	Provincial
a_0	0.957624	0.929037	0.940695
a_1	0.946740	0.967718	0.957211
a_2	1.000452	0.998511	0.999640
b_1	1.430462	1.236597	1.395784
b_2	-0.356702	-0.308204	-0.344672
b_3	2.950725	2.535507	2.835917
b_4	-1.455471	-1.222060	-1.396460
b_5	0.154263	0.146243	0.152487
p	0.225	0.225	0.225
n	2829	2894	5723
R^2	0.9803	0.9804	0.9807
MSE	0.8314	0.6239	0.7315

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

[a] See Appendix 1 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table 4. Coefficients for the taper model for balsam fir

Estimates	Natural region[a]		
	1 to 6, 9, 11 to 16	7, 8, 10	Provincial
a_0	0.918647	1.108006	1.002016
a_1	0.990225	0.898380	0.944076
a_2	0.997292	1.001816	0.999921
b_1	1.568514	1.338336	1.336330
b_2	-0.384262	-0.304630	-0.320352
b_3	3.503466	2.694363	2.839497
b_4	-1.677185	-1.277617	-1.324815
b_5	0.128169	0.087438	0.077452
p	0.225	0.225	0.225
n	1096	2016	3112
R^2	0.9839	0.9792	0.9803
MSE	1.0419	1.4907	1.4176

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

[a] See Appendix 1 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table 5. Coefficients for the taper model for aspen

Estimates	Natural region[a]				
	2, 14, 15, 16	9, 11	7, 8, 10	1, 3, 4, 5, 6, 12, 13	Provincial
a_0	0.944522	0.905615	0.588838	0.841897	0.790406
a_1	0.938030	0.964894	1.161895	0.997064	1.026943
a_2	1.001644	1.000054	0.992096	0.998713	0.997524
b_1	0.695363	0.553236	0.709300	0.536865	0.600584
b_2	-0.067849	-0.049737	-0.075446	-0.064020	-0.065681
b_3	0.050603	-0.280768	-0.116041	-0.234471	-0.173812
b_4	-0.016330	0.170687	0.040949	0.179963	0.121363
b_5	0.116432	0.075789	0.113638	0.031550	0.063253
p	0.225	0.225	0.225	0.225	0.225
n	2475	7932	2474	14968	27848
R^2	0.9848	0.9804	0.9791	0.9806	0.9804
MSE	0.9788	1.7208	1.9219	1.5698	1.6312

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

[a] See Appendix 1 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table 6. Coefficients for the taper model for balsam poplar

Estimates	Natural region[a]		
	7, 8, 9, 10, 11, 14	1 to 6, 12, 13, 15, 16	Provincial
a_0	0.913329	0.804370	0.861179
a_1	0.922590	0.982874	0.951483
a_2	1.002574	0.999527	1.000957
b_1	0.308448	0.996958	0.752581
b_2	-0.065670	-0.223248	-0.167305
b_3	-0.102130	1.106731	0.693611
b_4	0.226336	-0.459817	-0.224137
b_5	0.023148	-0.003392	0.008214
p	0.225	0.225	0.225
n	1680	1790	3470
R^2	0.9817	0.9710	0.9751
MSE	1.1578	2.6792	1.9692

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

[a] See Appendix 1 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table 7. Coefficients for the taper model for white spruce

Estimates	Natural region[a]			
	7, 8, 10	1 to 6, 12, 13, 15, 16	9, 11, 14	Provincial
a_0	0.713393	0.903528	0.862685	0.860438
a_1	1.071533	0.975136	0.993148	0.995406
a_2	0.996067	0.999018	0.998773	0.998493
b_1	1.153679	0.846981	1.135018	1.040218
b_2	-0.283807	-0.244969	-0.252377	-0.252387
b_3	2.022713	1.783097	1.885321	1.842818
b_4	-0.953783	-0.730236	-0.921437	-0.852227
b_5	0.101608	0.040997	0.150228	0.110359
p	0.225	0.225	0.225	0.225
n	2853	7945	10005	20803
R^2	0.9767	0.9825	0.9852	0.9831
MSE	4.0306	1.8197	2.1530	2.3370

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

[a] See Appendix 1 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table 8. Coefficients for the taper model for lodgepole pine

Estimates	Natural region[a]				
	7, 8	6, 9, 11, 14	4, 10	1, 2, 3, 5, 12, 13, 15, 16	Provincial
a_0	0.800648	0.957164	0.828665	1.033572	0.897617
a_1	1.053544	0.959992	1.024196	0.913621	0.988518
a_2	0.995568	0.999774	0.997492	1.000765	0.998735
b_1	0.568347	0.766747	0.596193	0.256633	0.675759
b_2	-0.125114	-0.140758	-0.118777	-0.049091	-0.130313
b_3	0.610085	0.666037	0.465591	-0.252118	0.570634
b_4	-0.238442	-0.355050	-0.196176	0.174267	-0.275457
b_5	0.045398	0.132140	0.083094	0.123722	0.105403
p	0.225	0.225	0.225	0.225	0.225
n	2042	7656	7376	743	17808
R^2	0.9733	0.9830	0.9817	0.9840	0.9823
MSE	1.1281	1.5764	1.3475	1.0140	1.4503

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

[a] See Appendix I for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table 9. Coefficients for the taper model for jack pine and other tree species

Estimates	Species				
	Jack pine	Douglas-fir	White birch	Tamarack	Engelmann spruce
a_0	0.940832	0.913153	0.894358	0.933517	1.072576
a_1	0.955575	0.964386	1.007721	0.965471	0.897766
a_2	0.999333	0.998391	0.991384	0.998393	1.001919
b_1	0.116311	1.386315	-0.483072	2.079455	1.301834
b_2	-0.028172	-0.286495	0.155593	-0.462028	-0.305439
b_3	-0.384427	1.783899	-2.273122	3.732057	2.265717
b_4	0.304055	-0.916932	1.326501	-1.950194	-1.119671
b_5	0.072192	0.058830	0.168897	0.190425	0.123519
p	0.225	0.225	0.225	0.225	0.225
n	3562	638	416	225	847
R^2	0.9828	0.9868	0.9828	0.9824	0.9790
MSE	1.0921	1.1199	0.4305	0.8189	2.5719

$$d = a_0 D^{a_1} a_2^{D} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5 (D/H)}$$

3.0 THE DIAMETER OUTSIDE/INSIDE BARK MODEL

The diameter outside/inside bark model takes the form of

$$DOB = a+bDIB$$

where:

DOB = diameter outside bark at any point on the stem (cm)

DIB = corresponding diameter inside bark on the stem (cm)

a, b = parameters to be estimated.

The coefficients of determination (R^2) and the mean squared errors (MSE) are computed by

$$R^2 = 1-\frac{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2}{\sum_{i=1}^{n}(y_i-\overline{y})^2}$$

and

$$MSE = \frac{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2}{n-m}$$

where:

y_i = observed diameter outside bark at any point on the stem

$\hat{y}_i$ = predicted diameter outside bark

$\bar{y}_i$ = observed average diameter outside bark

n = number of observations

m = number of parameters (m = 2).

Table 10. Coefficients for the diameter outside/inside bark model

Species	Natural regions[1]	Estimated coefficients a	Estimated coefficients b	n	R^2	MSE
Softwood group	2, 15, 16	0.496341	1.025974	795	0.9984	0.1316
	9, 11, 14	0.447477	1.024646	17196	0.9986	0.1836
	7, 8, 10	0.337955	1.029211	15685	0.9986	0.1482
	1, 3, 4, 5, 6, 12, 13	0.355429	1.031969	13685	0.9987	0.1224
Hardwood group	2, 14, 15, 16	0.052537	1.084738	3300	0.9969	0.2261
	9, 11	0.161950	1.073830	8425	0.9972	0.2918
	7, 8, 10	0.262986	1.072787	2214	0.9974	0.2788
	1, 3, 4, 5, 6, 12, 13	0.052846	1.085439	17220	0.9964	0.3451
Aspen	2, 14, 15, 16	0.024127	1.081189	2476	0.9976	0.1819
	9, 11	0.135261	1.072734	7612	0.9976	0.2444
	7, 8, 10	0.211340	1.073573	2474	0.9977	0.2471
	1, 3, 4, 5, 6, 12, 13	0.061755	1.079512	15016	0.9974	0.2407
	Provincial	0.091433	1.077082	27582	0.9976	0.2397
Balsam/alpine fir	7, 8, 10	0.323616	1.050716	2016	0.9975	0.2007
	1 to 6, 9, 11 to 16	0.249035	1.050236	1096	0.9983	0.1228
	Provincial	0.289940	1.051003	3125	0.9979	0.1751
Balsam poplar	7, 8, 9, 10, 11, 14	0.257085	1.103150	1414	0.9952	0.3947
	1 to 6, 12, 13, 15, 16	0.109731	1.125120	1790	0.9947	0.6157
	Provincial	0.149322	1.117988	3204	0.9948	0.5326
Lodgepole pine	7, 8	0.240347	1.020105	2046	0.9991	0.0416
	6, 9, 11, 14	0.294015	1.024582	7307	0.9991	0.0903
	4, 10	0.308258	1.024549	7407	0.9990	0.0771
	1, 2, 3, 5, 12, 13, 15, 16	0.189744	1.046810	743	0.9963	0.2572
	Provincial	0.283173	1.025305	17608	0.9990	0.0905
Black spruce	7, 8, 9, 10, 11	0.414614	1.030781	2829	0.9985	0.0663
	1 to 6, 12, 13, 14, 15, 16	0.349765	1.036689	2894	0.9982	0.0622
	Provincial	0.382746	1.033405	5723	0.9984	0.0646
White spruce	9, 11, 14	0.536767	1.022700	9681	0.9993	0.1122
	7, 8, 10	0.521645	1.024172	2853	0.9994	0.1161
	1 to 6, 12, 13, 15, 16	0.413577	1.028342	7955	0.9992	0.0851
	Provincial	0.484768	1.024893	20473	0.9993	0.1033
White birch	Provincial	0.077197	1.062798	416	0.9976	0.0666
Douglas-fir	Provincial	-0.095253	1.123153	661	0.9945	0.5852
Tamarack	Provincial	0.378870	1.034008	225	0.9984	0.0795
Jack pine	Provincial	0.161727	1.045672	3563	0.9949	0.3570
Engelmann spruce	Provincial	0.461270	1.023752	848	0.9993	0.0898

$DOB = a + bDIB$

[1] See Appendix 1 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

4.0 THE HEIGHT-DIAMETER MODEL

The height-diameter model takes the form of

$$H = 1.3 + a(1 - e^{-bD})^{c}$$

where:

H = total tree height (m)

D = diameter at breast height outside bark (cm)

e = base of the natural logarithm (≈ 2.71828)

a, b and c = parameters to be estimated.

A more detailed description of the development of the height-diameter equations is presented in Ecologically Based Individual Tree Height-Diameter Models for Major Alberta Tree Species.

The coefficients of determination (R^2) and the mean squared errors (MSE) are computed by

$$R^2 = 1 - \frac{\sum_{i=1}^{n} w_i (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} w_i (y_i - \bar{y})^2}$$

and

$$MSE = \frac{\sum_{i=1}^{n} w_i (y_i - \hat{y}_i)^2}{n - m}$$

where:

y_i = actual tree height

$\hat{y}_i$ = predicted tree height

$\bar{y}_i$ = observed average tree height

n = number of observations

m = number of parameters ($m = 3$)

$w_i = 1/D_i$.

Table 11. Coefficients for the provincial height-diameter model

Species	Estimated coefficients			n	R^2	MSE
	a	b	c			
Aspen	25.6614	0.06834	1.1394	3604	0.8734	0.3083
White birch	27.9727	0.03522	0.8695	101	0.8565	0.3301
Balsam/alpine fir	24.7532	0.06615	1.5695	497	0.9316	0.2662
Douglas-fir	21.3299	0.06090	1.5973	78	0.7912	0.1679
Tamarack	26.3266	0.05375	1.4026	39	0.8651	0.4101
Balsam poplar	25.5700	0.05050	0.9865	528	0.8067	0.3219
Jack pine	31.4263	0.03888	1.1279	589	0.9181	0.2669
Lodgepole pine	29.0075	0.04859	1.1782	3096	0.7873	0.3599
Black spruce	24.5751	0.05432	1.2243	1570	0.8647	0.2468
Engelmann spruce	36.3184	0.02604	1.0930	153	0.7732	0.3271
White spruce	32.1261	0.04633	1.3032	2889	0.8762	0.4214

$H = 1.3 + a(1 - e^{-bD})^{c}$

Table 12. Coefficients for the natural region based height-diameter model

Species	Natural regions[1]	Estimated coefficients a	b	c	n	R^2	MSE
Softwood group	2, 15, 16	30.7738	0.06562	1.6975	89	0.8831	0.3858
	9, 11, 14	32.4540	0.04648	1.3224	2828	0.8187	0.3905
	7, 8, 10	28.4311	0.04513	1.1839	3399	0.9126	0.3049
	1, 3, 4, 5, 6, 12, 13	31.9247	0.04372	1.2310	2594	0.8155	0.3722
Hardwood group	2, 14, 15, 16	27.1014	0.05186	0.9954	410	0.8155	0.3722
	9, 11	25.8069	0.06818	1.2063	1320	0.8491	0.3111
	7, 8, 10	27.7784	0.05235	1.3156	363	0.7094	0.3981
	1, 3, 4, 5, 6, 12, 13	24.6591	0.07797	1.2017	2140	0.9043	0.2697
Aspen	2, 14, 15, 16	26.5484	0.05699	0.9846	300	0.8688	0.2755
	9, 11	25.6731	0.07367	1.2608	1100	0.8701	0.2877
	7, 8, 10	28.0750	0.04860	1.2173	386	0.7073	0.4187
	1, 3, 4, 5, 6, 12, 13	24.8408	0.08081	1.2405	1836	0.9136	0.2400
Balsam/alpine fir	7, 8, 10	24.3383	0.06707	1.5909	252	0.9570	0.1798
	1 to 6, 9, 11 to 16	28.6319	0.05226	1.4467	161	0.9118	0.3496
Balsam poplar	7, 8, 9, 10, 11, 14	25.1413	0.06488	1.3192	206	0.7143	0.3361
	1 to 6, 12, 13, 15, 16	25.3810	0.05010	0.9270	236	0.8747	0.2840
Lodgepole pine	7, 8	24.4114	0.03555	0.7846	320	0.5534	0.2690
	6, 9, 11, 14	29.6276	0.05461	1.2997	1080	0.8217	0.2860
	4, 10	24.8398	0.06468	1.2937	1602	0.7708	0.3666
	1, 2, 3, 5, 12, 13, 15, 16	23.9518	0.07865	1.4813	94	0.8712	0.2302
Black spruce	7, 8, 9, 10, 11	24.9305	0.05281	1.2552	1037	0.8660	0.2465
	1 to 6, 12, 13, 14, 15, 16	24.3666	0.05775	1.2313	617	0.8737	0.2372
White spruce	9, 11, 14	32.4278	0.05055	1.3940	1185	0.8801	0.3681
	7, 8, 10	38.3117	0.02635	1.1152	526	0.8614	0.4580
	1 to 6, 12, 13, 15, 16	29.8812	0.05557	1.3911	1176	0.9020	0.3339

$$H = 1.3 + a(1 - e^{-bD})^{c}$$

[1] See Appendix 1 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

5.0 THE STUMP DIAMETER AND BREAST HEIGHT DIAMETER MODEL

The stump diameter and breast height diameter model takes form of

$$DOB_{stp} = a+bD+cD^2$$

where:

DOB_{stp} = stump diameter outside bark (cm) at 0.3 m stump height

D = diameter at breast height outside bark (cm)

a, b and c = parameters to be estimated.

The coefficients of determination (R^2) and the mean squared errors (MSE) are computed by

$$R^2 = 1-\frac{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2}{\sum_{i=1}^{n}(y_i-\overline{y})^2}$$

and

$$MSE = \frac{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2}{n-m}$$

where:

y_i = actual stump diameter outside bark

$\hat{y}_i$ = predicted stump diameter outside bark

$\bar{y}_i$ = observed average stump diameter outside bark

n = number of observations

m = number of parameters (m = 3).

Table 13. Coefficients for the stump diameter and breast height diameter model

Species	Natural region[1]	Estimated coefficients a	b	c	n	R^2	MSE
Softwood group	2, 15, 16	1.065686	1.006206	0.002586	85	0.9811	1.9421
	9, 11, 14	0.499206	1.044508	0.002113	1752	0.9735	4.4462
	7, 8, 10	-0.710193	1.127362	0.000588	1874	0.9742	3.5347
	1, 3, 4, 5, 6, 12, 13	-0.233569	1.119301	0.000390	1704	0.9828	2.4142
Hardwood group	2, 14, 15, 16	0.487002	0.990354	0.003679	409	0.9840	1.4813
	9, 11	0.095438	1.068140	0.001468	984	0.9753	3.0927
	7, 8, 10	0.007333	1.091470	0.001121	251	0.9716	3.0350
	1, 3, 4, 5, 6, 12, 13	0.047136	1.072125	0.001866	1967	0.9774	3.1221
Aspen	2, 14, 15, 16	0.114041	1.016604	0.003272	300	0.9852	1.4862
	9, 11	-0.093612	1.083466	0.001270	815	0.9762	3.2255
	7, 8, 10	-1.646520	1.239705	-0.001720	225	0.9753	3.2291
	1, 3, 4, 5, 6, 12, 13	-0.052028	1.076457	0.001805	1691	0.9774	3.0374
	Provincial	-0.330572	1.106639	0.000986	3099	0.9780	2.9946
Balsam/alpine fir	7, 8, 10	1.265808	0.952207	0.003527	252	0.9756	2.2304
	1 to 6, 9, 11 to 16	0.664028	1.019720	0.002754	161	0.9833	1.6215
	Provincial	1.028869	0.979696	0.003078	413	0.9802	2.0308
Balsam poplar	7, 8, 9, 10, 11, 14	0.587071	1.070632	0.000464	176	0.9756	2.0649
	1 to 6, 12, 13, 15, 16	0.762353	1.017031	0.002918	211	0.9754	3.8825
	Provincial	0.671062	1.037718	0.002077	387	0.9749	3.1733
Lodgepole pine	7, 8	-0.159741	1.046160	0.001804	280	0.9711	1.2603
	6, 9, 11, 14	0.582463	1.034249	0.001970	706	0.9711	2.9597
	4, 10	-0.487166	1.112282	0.000347	831	0.9753	2.0865
	1, 2, 3, 5, 12, 13, 15, 16	-0.321525	1.175896	-0.001650	91	0.9809	1.7527
	Provincial	-0.245285	1.088419	0.000993	1929	0.9762	2.3222
Black spruce	7, 8, 9, 10, 11	0.982793	0.943279	0.006344	439	0.9676	1.8523
	1 to 6, 12, 13, 14, 15, 16	0.125198	1.058588	0.002137	481	0.9767	0.9909
	Provincial	0.536125	0.996309	0.004617	920	0.9725	1.4253
White spruce	9, 11, 14	0.190577	1.069563	0.001822	892	0.9700	6.0680
	7, 8, 10	-2.112193	1.275528	-0.001564	298	0.9677	8.6912
	1 to 6, 12, 13, 15, 16	-0.582516	1.156757	-0.000271	879	0.9806	3.1438
	Provincial	-0.567783	1.142153	0.000429	2069	0.9750	5.3164
White birch	Provincial	0.300399	1.157729	-0.001896	71	0.9760	0.7442
Douglas-fir	Provincial	-1.689559	1.380771	-0.004699	80	0.9738	2.8534
Tamarack	Provincial	0.499971	1.065686	0.002899	34	0.9868	0.9720
Jack pine	Provincial	0.211020	1.144627	-0.001614	519	0.9794	2.1163
Engelmann spruce	Provincial	-0.107514	1.101339	0.001312	107	0.9732	4.6559

$$DOB_{stp} = a+bD+cD^2$$

[1] See Appendix 1 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Appendix 1.

List of Natural Regions of Alberta

Natural region 1 — Central mixedwood

Natural region 2 — Dry mixedwood

Natural region 3 — Wetland mixedwood

Natural region 4 — Sub-Arctic

Natural region 5 — Peace River Lowlands

Natural region 6 — Boreal Highlands

Natural region 7 — Alpine

Natural region 8 — Sub-Alpine

Natural region 9 — Montane

Natural region 10 — Upper Foothills

Natural region 11 — Lower Foothills

Natural region 12 — Athabasca Plain

Natural region 13 — Kazan Upland

Natural region 14 — Foothills Parkland

Natural region 15 — Peace River Parkland

Natural region 16 — Central Parkland

Natural region 17 — Dry mixedgrass

Natural region 18 — Foothills Fescue

Natural region 19 — Northern Fescue

Natural region 20 — Mixedgrass

Appendix 2.

List of Major Alberta Tree Species and Their Species Code

SPECIES	SPECIES CODE	SCIENTIFIC NAME
White spruce	Sw	*Picea glauca* (Moench) Voss
Tamarack	Lt	*Larix laricina* (Du Roi) K. Koch
Engelmann spruce	Se	*Picea engelmannii* Parry ex Engelm.
Lodgepole pine	Pl	*Pinus contorta* var. *latifolia* Engelm.
Jack pine	Pj	*Pinus banksiana* Lamb.
Aspen	Aw	*Populus tremuloides* Michx.
White birch	Bw	*Betula papyrifera* Marsh.
Balsam poplar	Pb	*Populus balsamifera* L.
Black spruce	Sb	*Picea mariana* (Mill.) B.S.P.
Balsam fir	Fb	*Abies balsamea* (L.) Mill.
Alpine fir	Fa	*Abies lasiocarpa* (Hook.) Nutt.
Douglas-fir	Fd	*Pseudotsuga menziesii* (Mirb.) Franco

Appendix 3.

Metric Conversion Chart

1 cm	= 0.39370 in.
1 m	= 3.28083 ft.
1 ha	= 2.47105 acres
1 m^2	= 10.76385 sq. ft.
1 m^3	= 35.31435 cu. ft
1 km	= 0.62137 miles
1 m^2/ha	= 4.3560 sq. ft/acre
1 m^3/ha	= 14.2913 cu. ft/acre

1 in.	= 2.5400 cm
1 ft.	= 0.3048 m
1 acre	= 0.4047 ha
1 sq. ft.	= 0.09290 m^2
1 cu. ft.	= 0.02832 m^3
1 mile	= 1.6093 km
1 fbm	= 1 ft. × 1 ft. × 1 in.
1 Mfbm	= 1000 foot board measure (fbm)

1 m^3 log	≈ 233 board feet lumber (provincial average conversion factor)
1 Mfbm	≈ 4.3 m^3 log (provincial average conversion factor)