Ecologically Based Individual Tree Volume Estimation for Major Alberta Tree Species

Report # 2

Ecologically Based Individual Tree Height-Diameter Models

for Major Alberta Tree Species

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4). Engelmann spruce (*Picea engelmannii* Parry ex Engelm.)
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ABSTRACT

Using the felled tree data, natural region based height-diameter equations were fitted for major Alberta tree species. Differences of the height-diameter relationships among natural regions are scrutinized using the extra sum of squares method. Natural regions of similar height-diameter relationships were combined to provide a composite model in order to facilitate the practical use of such relationships. Provincial height-diameter equations ignoring the differences among natural regions were also fitted to meet the need for making height predictions on a large provincial basis.

1.0 INTRODUCTION

This report presents provincial and natural region based height-diameter models for major Alberta tree species. The models are proposed to be used for predicting individual tree height from field measurement of tree diameter at breast height outside bark. While natural region based height-diameter models will generally provide more accurate height estimates on a regional basis, either natural region or provincial-based height-diameter models can be applied depending on the objectives of the user. Provincial-based height-diameter models ignore the differences among natural regions, and are appropriate for making height predictions on a provincial basis.

A more detailed description of the development of the height-diameter models is presented in Appendix 1 of this report: Development of Ecologically Based Height-Diameter Models for Major Alberta Tree Species.

2.0 THE HEIGHT-DIAMETER MODEL

The following height-diameter model was found appropriate for major Alberta tree species:

$$H = 1.3 + a(1 - e^{-bD})^{c}$$

Where: H = total tree height (m),

D = diameter at breast height outside bark (cm),

e = base of the natural logarithm (≈ 2.71828),

a, b, and c = parameters to be estimated,

1.3 = a constant used to reflect that when $D = 0$, $H = 1.3$.

The height-diameter model was fitted using the provincial felled-tree data. Estimated coefficients according to natural regions or for the whole province are listed in Tables 1 and 2, respectively. The coefficients of determination (R^2) and the mean squared errors (MSE) in the Tables are computed according to the following formulas:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} w_i (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} w_i (y_i - \bar{y})^2}$$

and

$$MSE = \frac{\sum_{i=1}^{n} w_i (y_i - \hat{y}_i)^2}{n - m}$$

where: y_i = actual tree height

$\hat{y}_i$ = predicted tree height

$\bar{y}_i$ = observed average tree height

n = number of observations

m = number of parameters ($m = 3$)

$w_i = 1/D_i$ (the weighting factor).

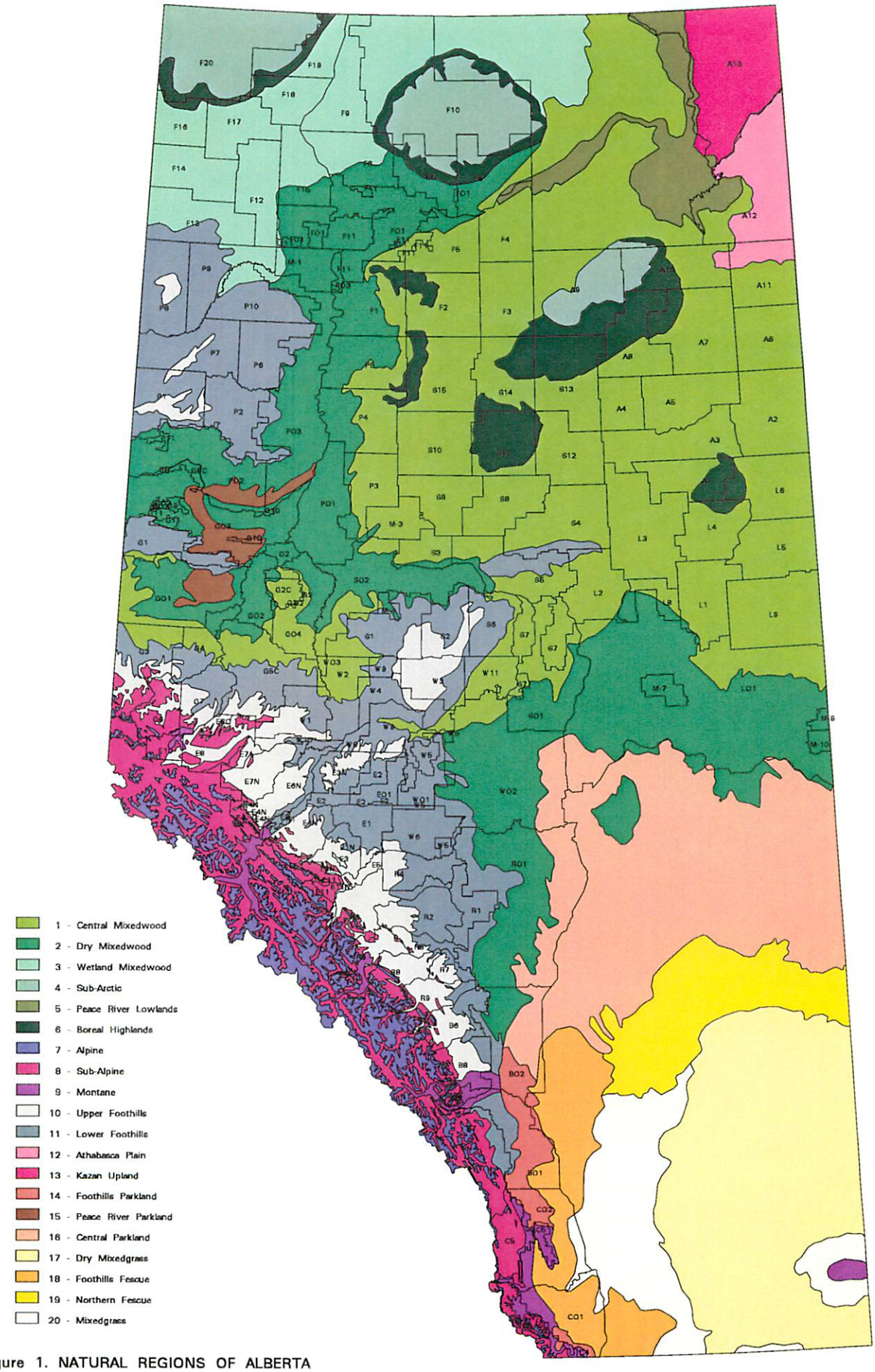

Figure 1. NATURAL REGIONS OF ALBERTA

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Scale 1:5 000 000

Table 1. Coefficients for the natural region based height-diameter model

Species	Natural regions[1]	Estimated coefficients			n	R^2	MSE
		a	b	c			
Softwood group	2, 15, 16	30.7738	0.06562	1.6975	89	0.8831	0.3858
	9, 11, 14	32.4540	0.04648	1.3224	2828	0.8187	0.3905
	7, 8, 10	28.4311	0.04513	1.1839	3399	0.9126	0.3049
	1, 3, 4, 5, 6, 12, 13	31.9247	0.04372	1.2310	2594	0.8155	0.3722
Hardwood group	2, 14, 15, 16	27.1014	0.05186	0.9954	410	0.8155	0.3722
	9, 11	25.8069	0.06818	1.2063	1320	0.8491	0.3111
	7, 8, 10	27.7784	0.05235	1.3156	363	0.7094	0.3981
	1, 3, 4, 5, 6, 12, 13	24.6591	0.07797	1.2017	2140	0.9043	0.2697
Aspen	2, 14, 15, 16	26.5484	0.05699	0.9846	300	0.8688	0.2755
	9, 11	25.6731	0.07367	1.2608	1100	0.8701	0.2877
	7, 8, 10	28.0750	0.04860	1.2173	386	0.7073	0.4187
	1, 3, 4, 5, 6, 12, 13	24.8408	0.08081	1.2405	1836	0.9136	0.2400
Balsam/alpine fir	7, 8, 10	24.3383	0.06707	1.5909	252	0.9570	0.1798
	1 to 6, 9, 11 to 16	28.6319	0.05226	1.4467	161	0.9118	0.3496
Balsam poplar	7, 8, 9, 10, 11, 14	25.1413	0.06488	1.3192	206	0.7143	0.3361
	1 to 6, 12, 13, 15, 16	25.3810	0.05010	0.9270	236	0.8747	0.2840
Lodgepole pine	7, 8	24.4114	0.03555	0.7846	320	0.5534	0.2690
	6, 9, 11, 14	29.6276	0.05461	1.2997	1080	0.8217	0.2860
	4, 10	24.8398	0.06468	1.2937	1602	0.7708	0.3666
	1, 2, 3, 5, 12, 13, 15, 16	23.9518	0.07865	1.4813	94	0.8712	0.2302
Black spruce	7, 8, 9, 10, 11	24.9305	0.05281	1.2552	1037	0.8660	0.2465
	1 to 6, 12, 13, 14, 15, 16	24.3666	0.05775	1.2313	617	0.8737	0.2372
White spruce	9, 11, 14	32.4278	0.05055	1.3940	1185	0.8801	0.3681
	7, 8, 10	38.3117	0.02635	1.1152	526	0.8614	0.4580
	1 to 6, 12, 13, 15, 16	29.8812	0.05557	1.3911	1176	0.9020	0.3339

$$H = 1.3 + a(1 - e^{-bD})^{c}$$

[1] See Appendix 2 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

Table 2. Coefficients for the provincial height-diameter model

Species	Estimated coefficients			n	R^2	MSE
	a	b	c			
Aspen	25.6614	0.06834	1.1394	3604	0.8734	0.3083
White birch	27.9727	0.03522	0.8695	101	0.8565	0.3301
Balsam/alpine fir	24.7532	0.06615	1.5695	497	0.9316	0.2662
Douglas-fir	21.3299	0.06090	1.5973	78	0.7912	0.1679
Tamarack	26.3266	0.05375	1.4026	39	0.8651	0.4101
Balsam poplar	25.5700	0.05050	0.9865	528	0.8067	0.3219
Jack pine	31.4263	0.03888	1.1279	589	0.9181	0.2669
Lodgepole pine	29.0075	0.04859	1.1782	3096	0.7873	0.3599
Black spruce	24.5751	0.05432	1.2243	1570	0.8647	0.2468
Engelmann spruce	36.3184	0.02604	1.0930	153	0.7732	0.3271
White spruce	32.1261	0.04633	1.3032	2889	0.8762	0.4214

$H = 1.3 + a(1 - e^{-bD})^{c}$

Appendix 1.

Development of Ecologically Based Height-Diameter Models for Major Alberta Tree Species

ABSTRACT

Using the felled tree data, natural region based height-diameter equations were fitted for major Alberta tree species. Differences of the height-diameter relationships among natural regions are scrutinized using the extra sum of squares method. Natural regions of similar height-diameter relationships were combined to provide a composite model in order to facilitate the practical use of such relationships. Provincial height-diameter equations ignoring the differences among natural regions were also fitted to meet the need for making height predictions on a large provincial basis.

INTRODUCTION

In a large number of forest inventories, total tree height is often predicted from observed tree diameter at breast height outside bark (DBH or D). Measurement of all sample trees for DBH and a subsample of trees that represents the range of diameters present for heights is an exceedingly common approach associated with both permanent or temporary sample plot systems and individual tree volume estimations. Using trees that have both height and diameter measured, a height-diameter relationship is developed. This relationship is then used to predict tree heights from field measurement of tree diameters.

A complete set of heights is frequently needed in estimation of tree volume and the top height statistics, as well as for the description of stands and their development over time (Arabatzis and Burkhart 1992; Arney 1985; Curtis 1967). Many growth and yield models also require height and diameter as two primary input variables, with all or part of the tree heights predicted from measured diameters (Arney

1985; Burkhart et al. 1972; Curtis et al. 1981; Wykoff et al. 1982). In some circumstances where the actual measurements of height growth are not available, height-diameter functions can also be used as a height increment model (Arney 1985; Larsen and Hann 1987). Determination of site index values may also require height-diameter equations for predictions of dominant and codominant heights.

With the increasing interests in ecology-based forest management in Alberta, facilitated by the revised and updated natural region classifications, there is a need to establish natural region based predictive relationships and to understand differences of these relationships among natural regions. The primary objectives of this study are to fit natural region based height-diameter equations for major Alberta tree species, and to compare the differences of the height-diameter relationships among natural regions. An appropriate height-diameter equation, which directly estimates tree height as a function of tree diameter, was first selected based on an evaluation of the relative performance of a variety of potential height-diameter functions on a large, regional data set covering numerous species. Natural region based height-diameter equations were then fitted and their differences scrutinized using appropriate statistical tests. Natural regions of similar height-diameter relationship were then combined to provide a single predictive relationship. Provincial height-diameter equations were also fitted to meet the need for making height predictions on a large provincial data basis.

THE DATA

Destructively sampled tree data for major Alberta tree species were obtained by Alberta Forest Service (AFS) personnel over the last three decades. Trees within a pre-selected variable-radius (prism point) plot or a circular fixed-area plot chosen according to certain specific criteria were felled and measured (Alberta Forest Service 1988). A prism was used to select individual trees with desired quality or characteristic in variable-radius plots, and every tree in circular fixed-area plots was selected.

Approximately 200 trees felled outside the permanent sample plots but inside the reservation boundary (buffer) were also included in this study. Trees that were not destructively measured for height

in the permanent sample plots were not used because of potential measurement errors. The total of 13,144 felled trees represent stands with a variety of densities, heights, species composition, stand structures, ages, and site conditions commonly found throughout the inventoried areas of the province. A detailed description of how the data are collected and recorded can be found in *Alberta Phase 3 Forest Inventory: Tree Sectioning Manual* (Alberta Forest Service 1988), and *Permanent Sample Plot Field Procedures Manual* (Alberta Forest Service 1993). The 13,144 trees include 12 different tree species. Appendix 3 lists the species, their scientific names, and the Alberta Forest Service species codes.

The original data set includes many different variables for individual trees and qualitative characteristics of their surround environment. Two variables available from the records, diameter at breast height outside bark and total tree height (H) for each tree, were selected to be used in this analysis. Provincial-based tree summary statistics including the mean, minimum, maximum, and standard deviation for total tree height and DBH by species are shown in Table 3. The variation in number of sample trees by species is an indication of relative importance. 165 dead trees and 415 trees that were forked, or had broken/dead tops were removed from the analysis.

Table 3. Tree summary statistics by species

Species	Number of trees	DBH (cm)				Height (m)			
		mean	min.	max.	std. dev.	mean	min.	max.	std. dev.
White spruce	2889	26.32	1.20	78.50	11.87	20.31	1.70	38.40	6.93
Tamarack	39	13.78	3.30	32.70	7.48	11.31	3.35	20.27	5.70
Engelmann spruce	153	23.46	6.50	50.30	9.83	16.01	5.40	30.50	5.57
Jack pine	589	17.92	1.60	45.00	9.96	14.72	2.58	27.13	6.50
Lodgepole pine	3096	22.00	1.10	64.60	8.42	18.19	1.72	36.80	5.16
Trembling aspen	3604	21.29	1.10	64.40	9.96	18.86	2.23	31.94	5.35
White birch	101	12.10	1.60	32.00	5.90	11.86	3.18	21.50	4.15
Balsam poplar	528	22.47	1.10	52.90	9.51	17.82	2.90	31.95	4.74
Black spruce	1570	13.98	1.10	55.30	5.96	12.18	1.76	30.63	4.18
Balsam/alpine fir	497	20.50	1.30	53.00	8.74	15.81	1.78	31.40	5.56
Douglas-fir	78	24.93	7.60	48.70	8.96	14.88	4.40	22.30	4.13

Provincial-based felled-tree data were further classified by natural regions of Alberta (Figure 1). Definitions for natural regions of Alberta are shown in Appendix 2. Qualitative variables such as township, range, and meridian were employed as classification criteria.

Since natural region based sample sizes for some species are either relatively small or concentrated in a few particular natural regions, only provincial-based models are considered for these species (white birch, tamarack, Douglas-fir, jack pine, and Engelmann spruce). Since there are only 16 alpine fir trees, they are combined with balsam fir in corresponding natural regions. Where appropriate, natural regions of the remaining tree species are combined into different natural region groups. Criteria for the grouping include number of observations, management objectives, and similarities of the height-diameter relationship pattern as revealed by plotting total tree height against DBH for various natural regions.

METHODS

Choice of Height-Diameter Model Form

A variety of potential height-diameter functions were selected for evaluation. The selection was based on the examination of height-diameter relationship as revealed by plotting total tree height against DBH for various species on a provincial basis. A complete list of the selected functions is shown in Table 4. They include those presented by Curtis (1967), Arabatzis and Burkhart (1992), Huang et al. (1992), and others. For this study, the polynomial-type height-diameter functions were not considered because extrapolation of the functions often leads to unrealistic height predictions.

Evaluation of the height-diameter functions follows the procedures as described by Huang et al. (1992). Results indicated that in terms of the fit of the functions for each species, several alternative functions may give very similar results and perform nearly equally well. However, judging from the mean squared errors, the asymptotic *t*-statistics of the parameters, the plots of studentized residuals against the predicted height, and the function's mathematical properties and its biological interpretations, as well as

Table 4. Height-diameter functions selected for evaluation

Function number and form	
[1] $H=1.3+aD^{b}$	[13] $H=1.3+a(1-e^{-bD^{c}})$
[2] $H=1.3+e^{a+b/(D+1)}$	[14] $H=1.3+ae^{-be^{-cD}}$
[3] $H=1.3+aD/(b+D)$	[15] $H=1.3+D^{2}/(a+bD+cD^{2})$
[4] $H=1.3+a(1-e^{-bD})$	[16] $H=1.3+aD^{bD^{-c}}$
[5] $H=1.3+D^{2}/(a+bD)^{2}$	[17] $H=1.3+ae^{b/(D+c)}$
[6] $H=1.3+ae^{b/D}$	[18] $H=1.3+a/(1+b^{-1}D^{-c})$
[7] $H=1.3+10^{a}D^{b}$	[19] $H=1.3+a(1-be^{-cD})^{d}$
[8] $H=1.3+aD/(D+1)+bD$	[20] $H=\left(y_1^{b}+(c^{b}-y_1^{b})\frac{[1-e^{-a(D-D_0)}]}{[1-e^{-a(D_2-D_0)}]}\right)^{\frac{1}{b}}$
[9] $H=1.3+a(D/(1+D))^{b}$	[21] $H=1.3+ae^{bD^{c}}$
[10] $H=1.3+e^{a+bD^{c}}$	[22] $H=1.3+e^{a}D^{b}$
[11] $H=1.3+a/(1+be^{-cD})$	[23] $H=1.3+e^{a+b/D}$
[12] $H=1.3+a(1-e^{-bD})^{c}$	

the principle of parsimony, the Chapman-Richards height-diameter function [12] was chosen:

$$H = 1.3+a(1-e^{-bD})^{c}$$

where:

H = total tree height (m),

D = diameter at breast height outside bark (cm),

e = base of the natural logarithm (≈ 2.71828),

a, b, and c = parameters to be estimated,

1.3 = a constant used to reflect that when $D = 0$, $H = 1.3$.

Comparison of Height-Diameter Models Among Natural Regions

To compare the differences of height-diameter models among natural regions, the regression method of dummy variables (also called indicator variables or binary variables) is used. Dummy variables are frequently applied to models that allow behavioral differences in geographic regions (Neter et al. 1990, Judge et al. 1988). For example, for the simple linear model $y = a + bx$, a dummy variable version of the model for two natural regions can be written as $y = (a + a_1x_1) + (b + b_1x_1)x$; this equation is the full model, where the dummy variable x_1 is defined as $x_1 = 0$ if natural region = 1, and $x_1 = 1$ if natural region = 2. It is obvious that the dummy variable version of the model represents two models: 1) for natural region 1 where $x_1 = 0$: $y = a + bx$ (the reduced model), and 2) for natural region 2 where $x_1 = 1$: $y = (a + a_1) + (b + b_1)x$. Identity of the two regression models for two natural regions is tested by considering the alternatives:

H_0: $a_1 = b_1 = 0$

H_a: not both $a_1 = 0$ and $b_1 = 0$

The appropriate test statistic, the extra sum of squares method (Neter et al. 1990), is given by

$$F^* = \frac{SSE(R) - SSE(F)}{df_R - df_F} \div \frac{SSE(F)}{df_F}$$

where F^* follows the F distribution when H_0 is true. The degrees of freedom df_R and df_F are associated with the reduced and the full model error sums of squares ($SSE(R)$ and $SSE(F)$), respectively. The statistical decision rule is:

If $F^* \leq F(1\text{-}\alpha;\ df_R - df_F,\ df_F)$, conclude H_o

If $F^* > F(1\text{-}\alpha;\ df_R - df_F,\ df_F)$, conclude H_a

The principles of dummy variables for linear least squares estimation can be readily extended to nonlinear models presented in this analysis. Condiser the nonlinear height-diameter model for aspen, if the purpose is to test the difference between natural regions 1 and 2, the dummy variable version of the full height-diameter model can be written as:

$$H = 1.3+(a+a_1x_1)\,[1-e^{-(b+b_1x_1)D}]^{(c+c_1x_1)}$$

This six-parameter full model has the following error sum of squares:

$$SSE(F) = SSE(a_1, b_1, c_1, a, b, c)$$

with $df_F = n - 6$ degrees of freedom associated with it. The dummy variable x_1 in the full model is defined as follows:

$$x_1 = \begin{matrix} 1 & \textit{if natural region} = 2 \\ 0 & \textit{otherwise} \end{matrix}$$

The reduced model for natural region group 1, for which $x_1 = 0$, is as follows:

$$H = 1.3+a(1-e^{-bD})^c$$

The error sum of squares for this three-parameter, reduced model is:

$$SSE(R) = SSE(a, b, c)$$

There are $df_R = n - 3$ degrees of freedom associated with this reduced model. Identity of the two height-diameter models for two natural region groups is tested by considering the alternatives:

H_0: $a_1 = b_1 = c_1 = 0$

H_a: at least one of the equalities in H_0 is not true

The test statistic in this case becomes:

$$F^* = \frac{SSE(a,b,c) - SSE(a_1,b_1,c_1,a,b,c)}{(n-3)-(n-6)} \div \frac{SSE(a_1,b_1,c_1,a,b,c)}{n-6}$$

To compute the test statistic here, both the full model and the reduced model are fitted to provide error sums of squares. Specifying the level of significance at 0.05, if the calculated $F^* \leq F$ (0.95; 3, n - 6), then H_o is true and the reduced model is appropriate for combined natural regions; if $F^* > F$ (0.95; 3, n - 6), then H_a is true and separate models are required for separate natural regions.

The test just described was conducted for each possible pair of natural regions, if the differences

of the height-diameter relationships are to be examined among three or more natural regions.

Parameter Estimation

Fitting of the full and the reduced models for various species was accomplished using the PROC NLIN procedure on SAS/STAT software (SAS Institute Inc. 1985). The derivative free iterative method DUD was applied, and multiple starting values for parameters were provided to ensure that the least squares solution was the global rather than the local solution. Because there is a common pattern of increasing variation as values of the dependent variable height increase, weighted nonlinear least squares was applied, with the weights selected to be inversely proportional to the variance of the error terms (Gallant 1987). A weighting factor of $w_i = 1/D_i$ was found most appropriate for all major tree species in Alberta (Huang et al. 1992). The nonlinear least squares estimator of the parameters is obtained by minimizing the weighted error sum of squares, which follows:

$$\sum_{i=1}^{n} w_i (H_i - \hat{H}_i)^2$$

where H_i and $\hat{H}_i$ are actual and predicted tree heights, and n is the number of observations. Because of the use of weighted nonlinear least squares method, the previously described F^* test uses weighted error sums of squares.

RESULTS AND DISCUSSION

Regional Difference of the Height-Diameter Models

For each species, regional difference of the height-diameter models is tested using the F^* statistic. With differences among natural regions properly identified, data from natural regions of similar height-diameter relationships were joined together, and a set of coefficients were estimated. Results of the classifications and estimates are displayed in Table 5. Natural region based individual tree height can be predicted using the estimated coefficients from Table 5.

Table 5. Coefficients for the natural region based height-diameter model

Species	Natural regions[1]	Estimated coefficients			n	R^2	MSE
		a	b	c			
Softwood group	2, 15, 16	30.7738	0.06562	1.6975	89	0.8831	0.3858
	9, 11, 14	32.4540	0.04648	1.3224	2828	0.8187	0.3905
	7, 8, 10	28.4311	0.04513	1.1839	3399	0.9126	0.3049
	1, 3, 4, 5, 6, 12, 13	31.9247	0.04372	1.2310	2594	0.8155	0.3722
Hardwood group	2, 14, 15, 16	27.1014	0.05186	0.9954	410	0.8155	0.3722
	9, 11	25.8069	0.06818	1.2063	1320	0.8491	0.3111
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Aspen	2, 14, 15, 16	26.5484	0.05699	0.9846	300	0.8688	0.2755
	9, 11	25.6731	0.07367	1.2608	1100	0.8701	0.2877
	7, 8, 10	28.0750	0.04860	1.2173	386	0.7073	0.4187
	1, 3, 4, 5, 6, 12, 13	24.8408	0.08081	1.2405	1836	0.9136	0.2400
Balsam/alpine fir	7, 8, 10	24.3383	0.06707	1.5909	252	0.9570	0.1798
	1 to 6, 9, 11 to 16	28.6319	0.05226	1.4467	161	0.9118	0.3496
Balsam poplar	7, 8, 9, 10, 11, 14	25.1413	0.06488	1.3192	206	0.7143	0.3361
	1 to 6, 12, 13, 15, 16	25.3810	0.05010	0.9270	236	0.8747	0.2840
Lodgepole pine	7, 8	24.4114	0.03555	0.7846	320	0.5534	0.2690
	6, 9, 11, 14	29.6276	0.05461	1.2997	1080	0.8217	0.2860
	4, 10	24.8398	0.06468	1.2937	1602	0.7708	0.3666
	1, 2, 3, 5, 12, 13, 15, 16	23.9518	0.07865	1.4813	94	0.8712	0.2302
Black spruce	7, 8, 9, 10, 11	24.9305	0.05281	1.2552	1037	0.8660	0.2465
	1 to 6, 12, 13, 14, 15, 16	24.3666	0.05775	1.2313	617	0.8737	0.2372
White spruce	9, 11, 14	32.4278	0.05055	1.3940	1185	0.8801	0.3681
	7, 8, 10	38.3117	0.02635	1.1152	526	0.8614	0.4580
	1 to 6, 12, 13, 15, 16	29.8812	0.05557	1.3911	1176	0.9020	0.3339

$$H = 1.3 + a(1 - e^{-bD})^{c}$$

[1] See Appendix 2 for list of natural regions and their designation numbers. Figure 1 shows the locations of natural regions.

The coefficients of determination (R^2) and the mean squared errors (MSE) in Table 5 are computed by the following formulas:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} w_i (H_i - \hat{H}_i)^2}{\sum_{i=1}^{n} w_i (H_i - \bar{H})^2}$$

and

$$MSE = \frac{\sum_{i=1}^{n} w_i (H_i - \hat{H}_i)^2}{n-p}$$

where:

H_i = actual tree height

$\hat{H}_i$ = predicted tree height

$\bar{H}$ = average actual tree height

$n - p$ = error degrees of freedom

p = number of parameters

$w_i = 1/D_i$.

Provincial Height-Diameter Models

Provincial height-diameter models were created for white birch, Douglas-fir, tamarack, jack pine, and Engelmann spruce. For species that have distinct height-diameter relationships by natural regions, use of appropriate natural region based models will generally provide more accurate height estimates. However, provincial-based models were also fitted for these species to accommodate the situations where such models were required. Results of the fit statistics are shown in Table 6. Fitted curves of the model for various species, along with the original height-diameter data, are displayed in Figures 2, 3 and 4.

Table 6. Coefficients for the provincial height-diameter model

Species	Estimated coefficients			n	R^2	MSE
	a	b	c			
Aspen	25.6614	0.06834	1.1394	3604	0.8734	0.3083
White birch	27.9727	0.03522	0.8695	101	0.8565	0.3301
Balsam/alpine fir	24.7532	0.06615	1.5695	497	0.9316	0.2662
Douglas-fir	21.3299	0.06090	1.5973	78	0.7912	0.1679
Tamarack	26.3266	0.05375	1.4026	39	0.8651	0.4101
Balsam poplar	25.5700	0.05050	0.9865	528	0.8067	0.3219
Jack pine	31.4263	0.03888	1.1279	589	0.9181	0.2669
Lodgepole pine	29.0075	0.04859	1.1782	3096	0.7873	0.3599
Black spruce	24.5751	0.05432	1.2243	1570	0.8647	0.2468
Engelmann spruce	36.3184	0.02604	1.0930	153	0.7732	0.3271
White spruce	32.1261	0.04633	1.3032	2889	0.8762	0.4214

$H = 1.3 + a(1 - e^{-bD})^c$

Table 6 and Figures 2 to 4 demonstrate that the Chapman-Richards height-diameter function performs well in depicting height-diameter relationships for major Alberta tree species. The function starts at a height of 1.3 metres when D equals zero, and is flexible enough to assume various shapes with different parameter values and to produce satisfactory curves under most circumstances. All the curves generated by the Chapman-Richards function assume biologically reasonable shapes that prevent unrealistic height predictions in cases where the functions are extrapolated beyond the range of the original data.

The most frequent application of the height-diameter equations fitted in this study is to fill in missing heights for trees that have no height measurement. This pertains to most forest inventories in which only a certain number of sample trees in a plot are measured for height. Given the diameter of the tree, and the coefficients from Table 5 or 6, height can be easily predicted with reasonable accuracy.

The height-diameter equations may also be used to smooth observations so as to achieve consistent height estimates under a regime of repeated remeasurements. It is necessary sometimes to revise height-

diameter equations every time when remeasurement data are available from permanent sample plots. Omule and Macdonald (1991) discussed procedures for doing this and showed that consistent height estimates could be obtained by constraining parameters of the height-diameter function. Arabatzis and Burkhart (1992) recommended that for updating existing height-diameter models, new trees be selected and measured on every occasion instead of retaining the same trees used previously for model fitting.

Development of distinct height-diameter equations at different measurement occasions from permanent sample plots was beyond the scope of this analysis. Interested readers may refer to Omule and Macdonald (1991), and Arabatzis and Burkhart (1992) for details. However, natural region- or provincial-based height-diameter equations developed in this study on large provincial stem analysis data sets representing diverse conditions typically found in the inventoried areas of Alberta can be used to approximate tree heights at any remeasurement occasion as long as tree diameters at these occasions are available.

CONCLUSIONS

Analysis of the natural region based height-diameter models for major Alberta tree species indicated that, while most of the height-diameter relationships were different among natural regions, there were a number of them that were very similar. Natural regions of similar height-diameter relationships were combined to give a composite model in order to facilitate the practical use of such relationships, while maximizing the accuracy of the height predictions. Use of the Chapman-Richards height-diameter function provided some of the most satisfactory fits among alternative model forms.

Provincial-based height-diameter models that ignore the differences among natural regions were also fitted for the purpose of making height predictions on a provincial basis. While natural region based models will generally provide more accurate height estimates on a regional basis, either natural region or provincial-based height-diameter models may be applied depending on the purposes of the users.

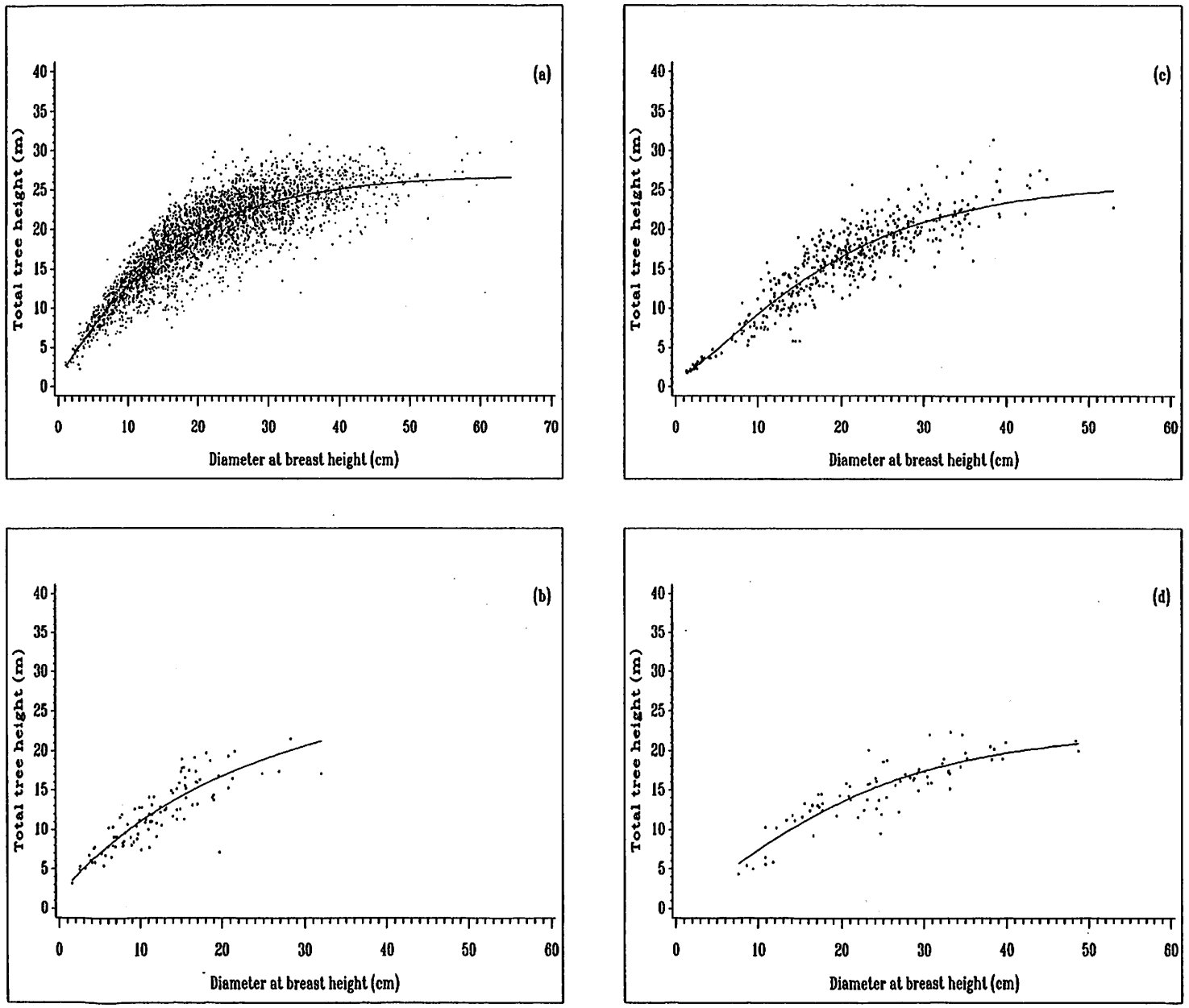

Figure 2. Plots of original height-diameter data and fitted curves for (a) aspen, (b) white birch, (c) balsam/alpine fir, and (d) Douglas-fir.

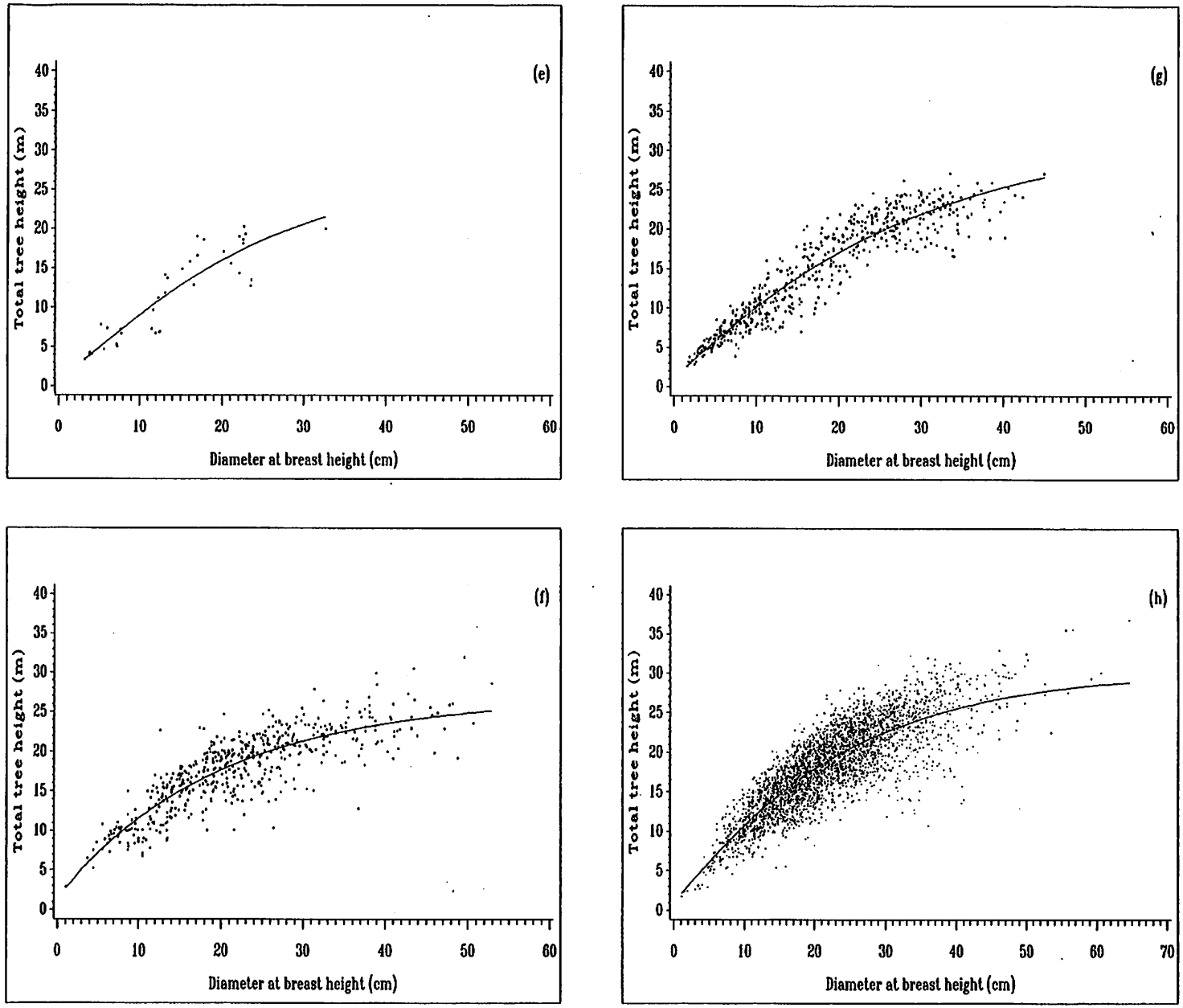

Figure 3. Plots of original height-diameter data and fitted curves for (e) tamarack, (f) balsam poplar, (g) jack pine, and (h) lodgepole pine.

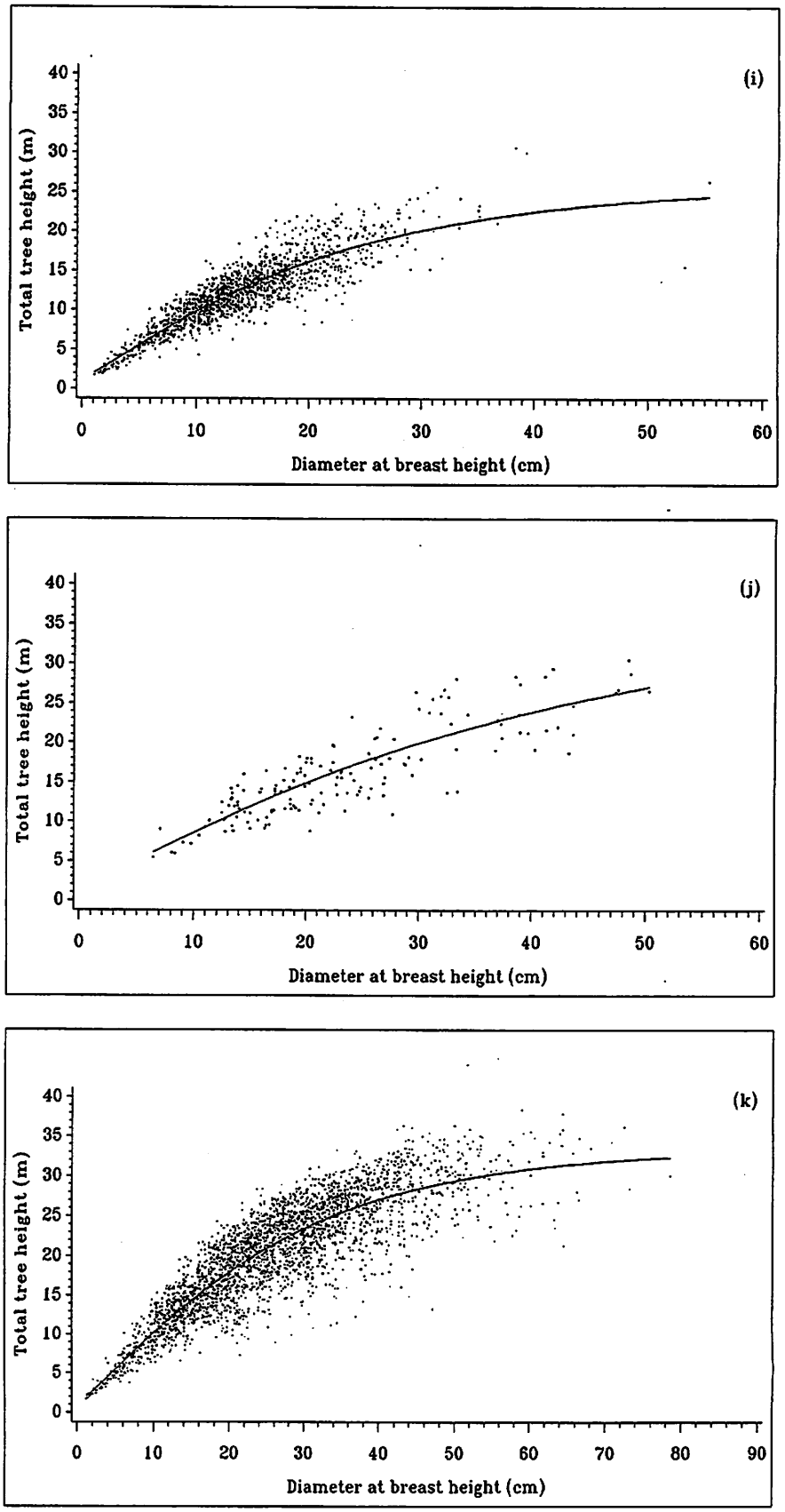

Figure 4. Plots of original height-diameter data and fitted curves for (i) black spruce, (j) Engelmann spruce, and (k) white spruce.

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Appendix 2.

List of Natural Regions of Alberta

Natural region 1 — Central mixedwood

Natural region 2 — Dry mixedwood

Natural region 3 — Wetland mixedwood

Natural region 4 — Sub-Arctic

Natural region 5 — Peace River Lowlands

Natural region 6 — Boreal Highlands

Natural region 7 — Alpine

Natural region 8 — Sub-Alpine

Natural region 9 — Montane

Natural region 10 — Upper Foothills

Natural region 11 — Lower Foothills

Natural region 12 — Athabasca Plain

Natural region 13 — Kazan Upland

Natural region 14 — Foothills Parkland

Natural region 15 — Peace River Parkland

Natural region 16 — Central Parkland

Natural region 17 — Dry mixedgrass

Natural region 18 — Foothills Fescue

Natural region 19 — Northern Fescue

Natural region 20 — Mixedgrass

Appendix 3.

List of Major Alberta Tree Species and Their Species Code

SPECIES	SPECIES CODE	SCIENTIFIC NAME
White spruce	Sw	*Picea glauca* (Moench) Voss
Tamarack	Lt	*Larix laricina* (Du Roi) K. Koch
Engelmann spruce	Se	*Picea engelmannii* Parry ex Engelm.
Lodgepole pine	Pl	*Pinus contorta* var. *latifolia* Engelm.
Jack pine	Pj	*Pinus banksiana* Lamb.
Aspen	Aw	*Populus tremuloides* Michx.
White birch	Bw	*Betula papyrifera* Marsh.
Balsam poplar	Pb	*Populus balsamifera* L.
Black spruce	Sb	*Picea mariana* (Mill.) B.S.P.
Balsam fir	Fb	*Abies balsamea* (L.) Mill.
Alpine fir	Fa	*Abies lasiocarpa* (Hook.) Nutt.
Douglas-fir	Fd	*Pseudotsuga menziesii* (Mirb.) Franco

Appendix 4.

Metric Conversion Chart

1 cm	= 0.39370 in.
1 m	= 3.28083 ft.
1 ha	= 2.47105 acres
1 m^2	= 10.76385 sq. ft.
1 m^3	= 35.31435 cu. ft
1 km	= 0.62137 miles
1 m^2/ha	= 4.3560 sq. ft/acre
1 m^3/ha	= 14.2913 cu. ft/acre
1 in.	= 2.5400 cm
1 ft.	= 0.3048 m
1 acre	= 0.4047 ha
1 sq. ft.	= 0.09290 m^2
1 cu. ft.	= 0.02832 m^3
1 mile	= 1.6093 km
1 fbm	= 1 ft. × 1 ft. × 1 in.
1 Mfbm	= 1000 foot board measure (fbm)
1 m^3 log	≈ 233 board feet lumber (provincial average conversion factor)
1 Mfbm	≈ 4.3 m^3 log (provincial average conversion factor)